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Anomalous gauge-boson couplings and the Higgs-boson mass

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Abstract. We study anomalous gauge-boson couplings induced by a locally $SU(2) \times U(1)$ invariant effective Lagrangian containing ten operators of dimension six built from boson fields of the standard model (SM) before spontaneous symmetry breaking (SSB). After SSB some operators lead to new three- and four-gauge-boson interactions, some contribute to the diagonal and off-diagonal kinetic terms of the gauge bosons, to the kinetic term of the Higgs boson and to the mass terms of the W and Z bosons. This requires a renormalisation of the gauge-boson fields, which, in turn, modifies the charged- and neutral-current interactions, although none of the additional operators contain fermion fields. Also the Higgs field must be renormalised. Bounds on the anomalous couplings from electroweak precision measurements at LEP and SLC are correlated with the Higgs-boson mass m_H. Rather moderate values of anomalous couplings allow m_H up to 500 GeV. At a future linear collider the triple-gauge-boson couplings γWW and ZWW can be measured in the reaction $e^+e^- \to WW$. We compare three approaches to anomalous gauge-boson couplings: the form-factor approach, the addition of anomalous-coupling terms to the SM Lagrangian after and, as outlined above, before SSB. The translation of the bounds on the couplings from one approach to another is not straightforward. We show that it can be done for the process $e^+e^- \to WW$ by defining new *effective* γWW and ZWW couplings.

Contents

1 Introduction

The standard model (SM) of particle physics has been tested in numerous aspects with impressive success. However, it lacks the attributes of a truly fundamental theory since it does not predict the number of particles or families and contains a large number of free parameters. Moreover, it does not incorporate gravity, so that ultimately a different theory has to replace the SM. One possibility is that physics beyond the SM will appear at an energy scale Λ. From current electroweak precision fits one estimates (see for instance [1]) that Λ should be at least of the order of TeV but, in fact, could be even much higher. The impact of this new high-scale physics on the phenomenology at lower energies can be taken into account in various ways.

In the form-factor (FF) approach the relevant vertices are parameterised in a general way. For the reaction $e^+e^- \to WW$ this was done in [2,3] for the three-gauge-boson vertices γWW and ZWW. There the structure of these two vertices is only restricted by Lorentz invariance. Form factors can and should have imaginary parts. Anomalous contributions to the γWW- and ZWW-form factors have been studied extensively both for LEP2 energies (see [4] and references therein) and for the energy range of future linear colliders [5–11].

Another possibility is to use an effective Lagrangian. Here we have two options. We can start from the SM Lagrangian *after* spontaneous symmetry breaking (SSB) and add terms of higher dimension to obtain an effective Lagrangian, which we call the ELa approach (effective Lagrangian after SSB). Alternatively we can start from the SM Lagrangian *before* SSB and add terms of higher dimension there, called the ELb approach (effective Lagrangian

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before SSB). In both cases the anomalous-coupling constants in the effective Lagrangian must be real. Anomalous imaginary parts in form factors are generated by loop effects using the effective-Lagrangian techniques familiar from chiral perturbation theory; see for instance [12]. The three approaches FF, ELa and ELb are related but should not be confused with each other; see the discussion in [13]. The ELa approach, taking the anomalous terms in leading order, produces only real parts of anomalous form factors. In the ELb approach the SSB has to be performed for the SM and the anomalous parts of the Lagrangian together. This has drastic consequences for all parts of the Lagrangian as we shall analyse in detail in this paper for various electroweak precision observables measured at LEP and SLC as well as for the reaction $e^+e^- \to WW$ at a future linear e^+e^- collider (LC). It also has the consequence that the counting of dimensions of anomalous terms is changed when Higgs fields are replaced by their vacuum expectation values; see [13], where also the question of $SU(2) \times U(1)$ gauge invariance is discussed. Anomalous couplings from operators of dimension n in the ELb approach will generate operators of dimension $n' \leq n$ in the ELa approach.

Some advantages and disadvantages of the three approaches are as follows. The FF approach is the most general one but it has the disadvantage of introducing many parameters. Also, the anomalous parts of form factors for different reactions like $e^+e^- \to WW$ and $\gamma\gamma \to WW$ are a priori not related. The ELa and ELb approaches allow one to relate anomalous effects in different reactions. Suppose now that we restrict the anomalous-coupling terms to dimension $n' \leq 6$ and $n \leq 6$ in the ELa and ELb approaches, respectively. Then the ELa approach generates more couplings than the ELb approach. Thus, in a sense, the ELb approach is the most restrictive framework if the dimension of the coupling terms is limited. For an application of the FF approach to the reaction $e^+e^- \to \tau^+\tau^-$ see for instance [14]; for an application of the ELa approach to Z decays see [15]. In the present paper we study mainly the ELb approach to anomalous electroweak gauge-boson couplings. We add to the SM Lagrangian – before SSB – operators of higher dimension that consist of SM fields. The natural expansion parameter for this series is (v/Λ), where $v \approx 246\,\text{GeV}$ is the vacuum expectation value of the SM-Higgs-boson field. Lists of all operators up to dimension six that respect the SM gauge symmetry $SU(3) \times SU(2) \times U(1)$ were given in [16,17]; see also references therein. A number of studies of the effects of these operators for phenomenology were made; see for instance [18,19]. We will comment below on the relation of these works to our present work. Here we follow [16] where systematic use is made of the equations of motion in order to reduce the number of operators to an independent set. A particularly interesting part of this Lagrangian is its gauge-boson sector because, in the SM, the structure of the gauge-boson vertices is highly restricted. In the SM there exist triple- as well as quartic-gauge-boson couplings all of which are fixed by the coupling constants of $SU(2)$ and $U(1)$; see for instance [20]. At tree level the triple couplings γWW, ZWW and only the quartic couplings $WWWW$, $\gamma\gamma WW$, γZWW and $ZZWW$ occur. Furthermore, in the SM the interactions of gauge bosons with the Higgs boson are determined by the covariant derivative acting on the Higgs field.

Here we consider the leading-order operators of dimension higher than four – that is of dimension six – that consist either only of electroweak gauge-boson fields or of gauge-boson fields combined with the Higgs-boson field of the SM. There are ten such operators, four of them CP violating [16]. This leads to ten new coupling constants h_i, subsequently called anomalous couplings, which parameterise deviations from the SM. It is assumed that the new-physics scale Λ is large enough such that operators of dimension six already give a good description of the high-scale effects. To keep the number of anomalous couplings within reasonable limits we exclude all non-SM operators that a priori involve fermions. Nevertheless, the purely bosonic anomalous couplings change the gauge-boson–fermion interactions in the following way. After SSB the pure boson operators contribute to the diagonal as well as off-diagonal kinetic terms of the gauge bosons and to the mass terms of the W and Z bosons. Firstly, this requires a renormalisation of the W-boson field. Secondly, the kinetic and the mass matrices of the neutral gauge bosons have to be diagonalised simultaneously to obtain the physical photon and Z-boson fields as linear combinations of the photon and Z-boson fields of the effective Lagrangian. This in turn modifies the neutral- and charged-current interactions. Since all fermion families are affected in the same manner no flavour-changing neutral currents are induced. Moreover two dimension-six operators contribute to the kinetic term of the Higgs boson such that a renormalisation of the Higgs field is necessary, too.

Thus in the ELb approach purely bosonic anomalous couplings influence also the precision observables from Z decay. In this paper we exploit this to calculate bounds on two CP conserving anomalous couplings from measurements at LEP1 and SLC and from W-boson measurements. To this end precision observables that are sensitive to the modified gauge-boson–fermion interactions or to the mass of the W boson are used. Less stringent bounds are obtained from direct measurements of the three-gauge-boson vertices γWW and ZWW in various processes at LEP2. However, one more CP conserving coupling and two CP violating couplings can be constrained using this data. Bounds on anomalous triple-gauge-boson couplings (TGCs) have been measured by the CDF collaboration [21] and the DØ collaboration [22] and are discussed in Sect. 6.1.

One important purpose of future high-energy experiments is the precision check of the relations between the various gauge-boson couplings. Their SM values guarantee the renormalisability of the electroweak theory. Thus any observed deviations from these SM values would have drastic consequences for the structure of the theory. Gauge-boson couplings can be studied at the LHC [23–25] and with high precision at a future LC like TESLA [26–28], NLC [8], JLC [29] or CLIC [30]. There W pair production, $e^+e^- \to WW$, is suitable to measure TGCs. In previous work [5,6,10,11] on $e^+e^- \to WW$ by our group we followed the form-factor approach using the parameterisation of the γWW and ZWW vertices of [3]. The maximum achievable sensitivity to the anomalous couplings in this process at CM energies of 500 GeV, 800 GeV and 3 TeV was determined by means of optimal observables [5,6] for the case of no or longitudinal beam polarisation in [10], and for transverse beam polarisation in [11]. Optimal observables were introduced for one-variable problems in [31] and for multi-variable problems in [5]. In the present paper we use, as explained above, the effective Lagrangian approach ELb. We give a detailed comparison of the FF and the ELb approaches for $e^+e^- \to WW$ in the following. In our ELb approach not only the γWW and ZWW vertices but also the gauge-boson–fermion vertices and the W and Z propagators get anomalous contributions. We show that nevertheless the results computed in the FF approach can be transformed into bounds on the anomalous couplings used here with ELb. This is achieved be defining new *effective* γWW and ZWW couplings that are specific for the reaction $e^+e^- \to WW$. In our ELb approach we have $SU(2) \times U(1)$ gauge invariance and we have restricted ourselves to dimension six for the additional operators. These two ingredients together lead to the well known "gauge relations" for the TGCs [4]. Note that $SU(2) \times U(1)$ gauge invariance alone gives no restrictions on the TGCs. Interestingly we find that even if the usual gauge relations hold for the original TGCs these relations change when we use the effective couplings, which are directly related to the FF approach. Moreover, even without *effective* couplings the shape of the gauge relations depends on the input parameter scheme.

In this paper we also mention some properties of the $\gamma\gamma WW$ and $\gamma\gamma H$ vertices that do not occur in the observables that we consider here but play an important rôle in the reaction $\gamma\gamma \to WW$ at a collider with two high-energy photons in the initial state. Such a photon collider has been proposed as an option for TESLA [32] and for CLIC [33]. The process $\gamma\gamma \to WW$ will be studied in forthcoming work [34]. Clearly, for a comparison of the reactions $e^+e^- \to WW$ and $\gamma\gamma \to WW$ the ELb framework is the most suitable one. This is the main motivation for treating $e^+e^- \to WW$ in the ELb approach in the present paper, since our results here are required for the discussion of $\gamma\gamma \to WW$ in [34]. There we shall give a comparison of the sensitivities of the reactions $e^+e^- \to WW$ and $\gamma\gamma \to WW$ to anomalous gauge-boson couplings.

This work is organised as follows: In Sect. 2 we give an overview of the operators in our effective Lagrangian (ELb approach) and explain, which operators contribute to the kinetic and mass terms of the gauge bosons and of the Higgs boson, to the three- and four-gauge-boson couplings, and to the photon–photon–Higgs coupling. In Sect. 3 we perform the simultaneous diagonalisation of the kinetic and mass terms of the neutral gauge bosons and the renormalisation of the charged gauge boson and Higgs boson fields. We then consider the interactions of gauge bosons with fermions in Sect. 4 and define two different sets of electroweak parameters, that we use to calculate the observables: one set, P_Z, containing the Z mass, the other one, P_W, containing the W mass. In Sect. 5 we present the bounds on the anomalous couplings from electroweak precision measurements at LEP and SLC, except for direct measurements of the three-gauge-boson vertices, thereby using P_Z. In Sect. 6 we give the relations of the standard couplings Δg_1^γ, $\Delta\kappa_\gamma$, etc. for the γWW and ZWW vertices to our anomalous couplings using P_Z and, alternatively, using P_W as input parameters. We derive bounds on the anomalous couplings of the effective Lagrangian from measurements of TGCs at LEP2 using P_Z. We analyse in detail the reaction $e^+e^- \to WW$ at a future LC where we define effective γWW and ZWW couplings using P_W. We calculate the bounds obtainable on the anomalous couplings using the results of [10,11] for this reaction. In Sect. 7 we present our conclusions.

2 Effective Lagrangian

Our starting point is the effective Lagrange density $\mathcal{L}_{\text{eff}}$ containing all lepton- and baryon-number-conserving operators that can be built from SM fields [16]. Let Λ be the scale of new physics and $v \approx 246$ GeV be the vacuum expectation value of the Higgs field. If not stated otherwise, numerical values of physical parameters are taken from [35]. Throughout this paper we assume

$$\Lambda \gg v. \tag{2.1}$$

Then $\mathcal{L}_{\text{eff}}$ can be expanded as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \dots\ , \tag{2.2}$$

where $\mathcal{L}_0$ contains operators of dimension less or equal to four, $\mathcal{L}_1$ of dimension five, $\mathcal{L}_2$ of dimension six etc. The terms $\mathcal{L}_1, \mathcal{L}_2, \dots$ give contributions of order (v/Λ), $(v/\Lambda)^2, \dots$ in the amplitudes, thus (2.2) represents effectively an expansion in powers of (v/Λ).

Given the SM particle content, the general form of $\mathcal{L}_0$ is fixed as that of the SM Lagrangian by gauge invariance. For the SM Lagrangian we use the conventions of [20]. Restricting ourselves to the electroweak interactions and neglecting neutrino masses we have (see Chap. 22 of [20])

$$\begin{aligned}\mathcal{L}_0 = &-\frac{1}{4}W^i_{\mu\nu}W^{i\,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ (\mathcal{D}_\mu\varphi)^\dagger\,(\mathcal{D}^\mu\varphi) + \mu^2\varphi^\dagger\varphi - \lambda\left(\varphi^\dagger\varphi\right)^2 \\ &+ \mathrm{i}\overline{L}\not{\mathcal{D}}L + \mathrm{i}\overline{E}\not{\mathcal{D}}E + \mathrm{i}\overline{Q}\not{\mathcal{D}}Q + \mathrm{i}\overline{U}\not{\mathcal{D}}U + \mathrm{i}\overline{D}\not{\mathcal{D}}D \\ &- \left(\overline{E}\,\Gamma_E\,\varphi^\dagger L + \overline{U}\,\Gamma_U\,\tilde{\varphi}^\dagger Q + \overline{D}\,\Gamma_D\,\varphi^\dagger Q + \text{H.c.}\right).\end{aligned} \tag{2.3}$$

The 3×3 Yukawa matrices have the form

$$\Gamma_E = \operatorname{diag}(c_\mathrm{e}, c_\mu, c_\tau), \tag{2.4}$$

$$\Gamma_U = \operatorname{diag}(c_u, c_c, c_t), \tag{2.5}$$

$$\Gamma_D = V\operatorname{diag}(c_d, c_s, c_b)V^\dagger, \tag{2.6}$$

where the diagonal elements all obey $c_i \geq 0$ and V is the CKM matrix. With these conventions the matrices Γ_E,

Table 1. Weak hypercharge of the fermions and the Higgs doublet

	L	E	Q	U	D	φ
y	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

Γ_U, Γ_D correspond to the matrices C_ℓ, C'_q, C_q in [20], respectively. The vector of the three left-handed lepton doublets is denoted by L, of the right-handed charged leptons by E, of the left-handed quark doublets by Q, and of the right-handed up- and down-type quarks by U and D. The Higgs field is denoted by φ and we define

$$\tilde{\varphi} = \varepsilon\varphi^*, \qquad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{2.7}$$

The covariant derivative is

$$\mathcal{D}_\mu = \partial_\mu + \mathrm{i}gW^i_\mu \mathbf{T}_i + \mathrm{i}g' B_\mu \mathbf{Y}, \tag{2.8}$$

where $\mathbf{T}_i$ and $\mathbf{Y}$ are the generating operators of weak-isospin and weak-hypercharge transformations. For the left-handed fermion fields and the Higgs doublet we have $\mathbf{T}_i = \tau_i/2$, where τ_i are the Pauli matrices. For the right-handed fermion fields we have $\mathbf{T}_i = 0$. The hypercharges y of the fermions and the Higgs doublet are listed in Table 1. The field strengths are[1]

$$\begin{aligned} W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\,\epsilon_{ijk}\, W^j_\mu W^k_\nu, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned} \tag{2.9}$$

For the parameters of the Higgs potential in (2.3) we assume

$$\mu^2 > 0, \qquad \lambda > 0. \tag{2.10}$$

Then the potential has a minimum for constant field satisfying

$$\sqrt{2\varphi^\dagger\varphi} = \sqrt{\frac{\mu^2}{\lambda}} \equiv v. \tag{2.11}$$

After SSB, that is in the unitary gauge, we can choose the Higgs field to have the form

$$\varphi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H'(x) \end{pmatrix}, \tag{2.12}$$

where $H'(x)$ would be the physical Higgs field in the SM, and in lowest order the vacuum expectation value of the Higgs field, v, is given in terms of the Lagrangian parameters by (2.11). Looking at the Higgs-mass term we find for the squared mass of the Higgs boson in the SM

$$m'^2_H = 2\lambda v^2. \tag{2.13}$$

[1] The signs in front of the gauge couplings in (2.8) and (2.9) differ from the conventions of [16]. This may lead to sign changes in the dimension-six operators discussed below.

The coupling constants in (2.4) to (2.6) are related to the fermion masses by

$$m_j = c_j \frac{v}{\sqrt{2}} \tag{2.14}$$

with $j = u, c, t, d, s, b, e, \mu, \tau$.

The higher-dimensional operators in $\mathcal{L}_1$, $\mathcal{L}_2$ etc. in (2.2) describe the effects of new physics at the scale Λ on the phenomenology at the weak scale v. Following [16,17], we assume $SU(3)\times SU(2)\times U(1)$ gauge invariance also for the new interactions. The only Lorentz and gauge invariant operator of dimension five that can be constructed from SM fields violates lepton-number conservation [16] and hence is not considered here. Thus, the leading-order addition to the SM Lagrangian is $\mathcal{L}_2$, which should therefore lead to a good description of the new-physics effects at energies sufficiently below Λ. Compared to [17] the number of operators of dimension six to be considered is reduced in [16] by systematically applying the equations of motion. This is a completely legitimate procedure for our purposes; see also the discussion of this point in [13]. We thus refer to the list of operators in [16] for our analysis.

Out of the 80 dimension-six operators listed in [16] we consider all operators that consist either only of electroweak gauge-boson fields or of gauge-boson fields combined with the SM Higgs field; see (3.5), (3.6) and (3.41) to (3.44) in [16]:

$$\begin{aligned} O_W &= \epsilon_{ijk}\, W^{i\,\nu}_\mu W^{j\,\lambda}_\nu W^{k\,\mu}_\lambda, \\ O_{\tilde{W}} &= \epsilon_{ijk}\, \tilde{W}^{i\,\nu}_\mu W^{j\,\lambda}_\nu W^{k\,\mu}_\lambda, \end{aligned} \tag{2.15}$$

$$\begin{aligned} O_{\varphi W} &= \frac{1}{2}\left(\varphi^\dagger\varphi\right)\, W^i_{\mu\nu} W^{i\,\mu\nu}, \\ O_{\varphi\tilde{W}} &= \left(\varphi^\dagger\varphi\right)\, \tilde{W}^i_{\mu\nu} W^{i\,\mu\nu}, \end{aligned} \tag{2.16}$$

$$\begin{aligned} O_{\varphi B} &= \frac{1}{2}\left(\varphi^\dagger\varphi\right)\, B_{\mu\nu} B^{\mu\nu}, \\ O_{\varphi\tilde{B}} &= \left(\varphi^\dagger\varphi\right)\, \tilde{B}_{\mu\nu} B^{\mu\nu}, \end{aligned} \tag{2.17}$$

$$\begin{aligned} O_{WB} &= \left(\varphi^\dagger\tau^i\varphi\right)\, W^i_{\mu\nu} B^{\mu\nu}, \\ O_{\tilde{W}B} &= \left(\varphi^\dagger\tau^i\varphi\right)\, \tilde{W}^i_{\mu\nu} B^{\mu\nu}, \end{aligned} \tag{2.18}$$

$$\begin{aligned} O^{(1)}_\varphi &= \left(\varphi^\dagger\varphi\right)\left(\mathcal{D}_\mu\varphi\right)^\dagger\left(\mathcal{D}^\mu\varphi\right), \\ O^{(3)}_\varphi &= \left(\varphi^\dagger\mathcal{D}_\mu\varphi\right)^\dagger\left(\varphi^\dagger\mathcal{D}^\mu\varphi\right). \end{aligned} \tag{2.19}$$

Here the dual field strengths are defined as

$$\tilde{W}^i_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} W^{i\,\rho\sigma}, \qquad \tilde{B}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} B^{\rho\sigma}. \tag{2.20}$$

In the following we therefore use the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \mathcal{L}_2, \tag{2.21}$$

where $\mathcal{L}_0$ is the SM part (2.3). The non-SM part with the dimension-six operators is

$$\mathcal{L}_2 = \Big(h_W O_W + h_{\tilde{W}} O_{\tilde{W}} + h_{\varphi W} O_{\varphi W} + h_{\varphi \tilde{W}} O_{\varphi \tilde{W}} + h_{\varphi B} O_{\varphi B} + h_{\varphi \tilde{B}} O_{\varphi \tilde{B}} + h_{WB} O_{WB} + h_{\tilde{W}B} O_{\tilde{W}B} + h_\varphi^{(1)} O_\varphi^{(1)} + h_\varphi^{(3)} O_\varphi^{(3)}\Big)/v^2, \tag{2.22}$$

where we have divided by v^2 in order to obtain dimensionless coupling constants h_i, with $i = W, \tilde{W}, \varphi W, \ldots$ The h_i are subsequently called anomalous couplings. Nominally we have

$$h_i = O(v^2/\Lambda^2). \tag{2.23}$$

3 Symmetry breaking and diagonalisation in the gauge-boson sector

Starting from the Lagrangian (2.21) we go now to the unitary gauge, that is we replace the Higgs field everywhere by the expression (2.12) involving only the Higgs-vacuum expectation value v and the field $H'(x)$, which would be the physical Higgs-boson field for zero anomalous couplings. If this is done for $\mathcal{L}_0$ we arrive at the SM Lagrangian in unitary gauge; see (22.123) of [20]. It is convenient to take this as starting point and consider the necessary changes due to the $\mathcal{L}_2$ term in (2.21) subsequently. Let us, therefore, introduce boson fields A'_μ, Z'_μ and $W'^{\pm}_\mu$ which would be the physical gauge-boson fields if we considered only the SM Lagrangian $\mathcal{L}_0$. The original W^i_μ and B_μ fields are expressed in terms of these fields as follows:

$$W^1_\mu = \frac{1}{\sqrt{2}}\left(W'^+_\mu + W'^-_\mu\right), \quad W^2_\mu = \frac{\mathrm{i}}{\sqrt{2}}\left(W'^+_\mu - W'^-_\mu\right), \tag{3.1}$$

$$W^3_\mu = c'_\mathrm{w}\, Z'_\mu + s'_\mathrm{w}\, A'_\mu, \quad B_\mu = -s'_\mathrm{w}\, Z'_\mu + c'_\mathrm{w}\, A'_\mu, \tag{3.2}$$

where

$$s'_\mathrm{W} \equiv \sin\theta'_\mathrm{W} = \frac{g'}{\sqrt{g^2 + g'^2}}, \tag{3.3}$$

$$c'_\mathrm{W} \equiv \cos\theta'_\mathrm{W} = \frac{g}{\sqrt{g^2 + g'^2}} \tag{3.4}$$

are the sine and cosine of the weak mixing angle in the SM, determined by the $SU(2)$ and $U(1)_Y$ couplings of $\mathcal{L}_0$. Without loss of generality we can assume g and g' to be greater than zero and therefore have $0 \le \theta'_\mathrm{W} \le \pi/2$. The positron charge e' of $\mathcal{L}_0$ is given by

$$e' = g s'_\mathrm{W}. \tag{3.5}$$

The next step is to consider the term $\mathcal{L}_2$ in (2.21) and (2.22), and insert for the Higgs field $\varphi(x)$ everywhere (2.12) and for the gauge-boson fields (3.1) and (3.2). We see then easily that the original dimension-six operators in $\mathcal{L}_2$ give now contributions to dimension-two, -three, -four, -five and -six terms.

In Table 2 we list from which coupling constants in (2.22) corresponding to the operators (2.15) to (2.19) we get contributions to the kinetic and mass terms of the gauge bosons, to the kinetic terms of the Higgs boson, and to several coupling terms of the gauge bosons and the Higgs boson in the basis $W'^{\pm}$, Z', A', H'. The kinetic terms of the gauge bosons receive contributions only from $O_{\varphi W}$, $O_{\varphi B}$ and O_{WB}. The operators $O_{\varphi\tilde{W}}$, $O_{\varphi\tilde{B}}$ or $O_{\tilde{W}B}$ do not contribute there since their terms of second order in the boson fields vanish after partial integration. The operators $O_\varphi^{(1)}$ and $O_\varphi^{(3)}$ contribute only to the gauge-boson-mass terms and to the kinetic term of the Higgs field H'.

In Table 2 we also show how the dimension-six operators contribute to those gauge-boson and gauge-boson–Higgs vertices that are required for our studies. Note that in Table 2 we show the contributions to the vertices where the operators are still written in terms of the primed fields $W'^{\pm}$, Z', A', H'. The operators O_W and $O_{\tilde{W}}$ contribute both to the three- and to the four-gauge-boson couplings. The operators $O_{\varphi W}$, O_{WB} and $O_{\tilde{W}B}$ contribute to the three-gauge-boson vertices with terms proportional to v^2. In addition, the operator $O_{\varphi W}$ also induces a four-gauge-boson vertex. The operator $O_{\varphi\tilde{W}}$ contributes neither to the TGCs, since the corresponding term can be written as a total divergence, nor to the four-gauge-boson couplings

Table 2. Contributions from SM Lagrangian and from operators (2.15) to (2.19) to kinetic and mass terms of gauge bosons, to the kinetic term of the Higgs boson and to terms of the form $V'W'^+W'^-$, $A'A'W'^+W'^-$ and $A'A'H$ with $V' = A'$ or Z'. Note that the contributions to the *physical* γWW, ZWW and $\gamma\gamma H'$ vertices *after* the simultaneous diagonalisation are different; see Table 7 below

	SM	h_W	$h_{\tilde{W}}$	$h_{\varphi W}$	$h_{\varphi\tilde{W}}$	$h_{\varphi B}$	$h_{\varphi\tilde{B}}$	h_{WB}	$h_{\tilde{W}B}$	$h_\varphi^{(1)}$	$h_\varphi^{(3)}$
gauge kinetic	√			√		√		√			
gauge mass	√									√	√
Higgs kinetic	√									√	√
$V'W'^+W'^-$	√	√	√	√				√	√		
$A'A'W'^+W'^-$	√	√	√	√							
$A'A'H'$				√	√	√	√	√	√		

because the term of the form

$$\epsilon^{\mu\nu\rho\sigma}\epsilon_{ijk}\epsilon_{ilm}W^j_\mu W^k_\nu W^l_\rho W^m_\sigma \tag{3.6}$$

vanishes for symmetry reasons. In addition, six operators give rise to a $A'A'H'$ vertex. The dimension-six operators of $\mathcal{L}_2$ induce anomalous terms to further vertices, e.g. $Z'Z'H'$ and $W'^+W'^-H'$, which are however not relevant for our calculations.

We see that with the inclusion of $\mathcal{L}_2$, the kinetic and the mass terms of the gauge bosons as well as the kinetic term of the Higgs field H' do not have standard form any more due to additional contributions arising according to Table 2. We have now to diagonalise the mass matrix and simultaneously transform the kinetic matrix to the unit matrix to identify the physical gauge-boson fields and the physical Higgs-boson field. The gauge-boson kinetic and mass terms of the effective Lagrangian (2.21) are given by

$$\mathcal{L}_V^{(2)} + \mathcal{L}_W^{(2)} , \tag{3.7}$$

where

$$\mathcal{L}_V^{(2)} = -\frac{1}{4}\boldsymbol{V}'^{\mathrm{T}}_{\mu\nu}\, T'\, \boldsymbol{V}'^{\mu\nu} + \frac{1}{2}\boldsymbol{V}'^{\mathrm{T}}_{\mu}\, M'\, \boldsymbol{V}'^{\mu} , \tag{3.8}$$

$$\mathcal{L}_W^{(2)} = -\frac{1}{2}\left(1-h_{\varphi W}\right) W'^+_{\mu\nu}W'^{-\mu\nu} + m'^2_W \left(1+h_\varphi^{(1)}/2\right) W'^+_\mu W'^{-\mu} , \tag{3.9}$$

$$\boldsymbol{V}'_{\mu\nu} = \partial_\mu \boldsymbol{V}'_\nu - \partial_\nu \boldsymbol{V}'_\mu , \quad \boldsymbol{V}'_\mu = \left(Z'_\mu, A'_\mu\right)^{\mathrm{T}} , \tag{3.10}$$

$$W'^\pm_{\mu\nu} = \partial_\mu W'^\pm_\nu - \partial_\nu W'^\pm_\mu . \tag{3.11}$$

Here we have introduced vector notation for the neutral primed gauge fields, and T' and M' are given by

$$T' = \begin{pmatrix} a & b \\ b & d \end{pmatrix} , \tag{3.12}$$

$$M' = m'^2_Z \left(1+\frac{1}{2}\left(h_\varphi^{(1)}+h_\varphi^{(3)}\right)\right)\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

with

$$a = 1 - 2c'_{\mathrm{W}}s'_{\mathrm{W}}h_{WB} - c'^2_{\mathrm{W}}h_{\varphi W} - s'^2_{\mathrm{W}}h_{\varphi B}, \tag{3.13}$$

$$b = \left(c'^2_{\mathrm{W}} - s'^2_{\mathrm{W}}\right) h_{WB} + c'_{\mathrm{W}}s'_{\mathrm{W}}\left(h_{\varphi B} - h_{\varphi W}\right) , \tag{3.14}$$

$$d = 1 + 2c'_{\mathrm{W}}s'_{\mathrm{W}}h_{WB} - s'^2_{\mathrm{W}}h_{\varphi W} - c'^2_{\mathrm{W}}h_{\varphi B}. \tag{3.15}$$

The quantities

$$m'^2_W = g^2v^2/4, \tag{3.16}$$

$$m'^2_Z = (g^2+g'^2)v^2/4 \tag{3.17}$$

would be the squared gauge-boson masses after SSB if we considered only the SM Lagrangian $\mathcal{L}_0$. Because of charge conservation there is no mixing between charged and neutral gauge-boson fields in (3.7). Moreover, the matrix M' has only one non-zero entry (corresponding to $Z'Z'$) since terms of second order in the gauge fields without derivatives can only come from operators with two covariant derivatives of Higgs fields, as occurring in (2.3) and (2.19). There, due to (2.12), only the massive gauge bosons contribute.

We would like to find a basis in the fields such that (3.7) takes the standard form:

$$\mathcal{L}_V^{(2)} = -\frac{1}{4}\left(Z_{\mu\nu}Z^{\mu\nu} + A_{\mu\nu}A^{\mu\nu}\right) + \frac{1}{2}m_Z^2 Z_\mu Z^\mu, \tag{3.18}$$

$$\mathcal{L}_W^{(2)} = -\frac{1}{2}W^+_{\mu\nu}W^{-\mu\nu} + m_W^2 W^+_\mu W^{-\mu}, \tag{3.19}$$

where

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu, \tag{3.20}$$

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{3.21}$$

$$W^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu, \tag{3.22}$$

and m_Z and m_W are (in lowest order) the physical masses of the Z and W bosons, respectively. For the charged fields this can be easily achieved by a rescaling

$$m_W^2 = \left(\frac{1+h_\varphi^{(1)}/2}{1-h_{\varphi W}}\right) m'^2_W = \left(\frac{1+h_\varphi^{(1)}/2}{1-h_{\varphi W}}\right)\frac{g^2v^2}{4}, \tag{3.23}$$

$$W^\pm_\mu = \sqrt{1-h_{\varphi W}}\; W'^\pm_\mu. \tag{3.24}$$

In the approximation linear in the anomalous couplings (3.23) agrees with (4.5a) in [16] (where the definition of v differs by a factor of $\sqrt{2}$ from ours) and with (3) in [17]. In the case of the neutral fields we perform a linear transformation

$$\boldsymbol{V}'_\mu = C\,\boldsymbol{V}_\mu, \tag{3.25}$$

where

$$\boldsymbol{V}_\mu = (Z_\mu, A_\mu)^{\mathrm{T}}. \tag{3.26}$$

Choosing the non-orthogonal matrix

$$C = \begin{pmatrix} \sqrt{d/t} & 0 \\ -b/\sqrt{dt} & 1/\sqrt{d} \end{pmatrix} \tag{3.27}$$

with $t = ad - b^2$, we obtain the desired form

$$T = C^{\mathrm{T}}T'C = \mathbb{1}, \qquad M = C^{\mathrm{T}}M'C = \begin{pmatrix} m_Z^2 & 0 \\ 0 & 0 \end{pmatrix}, \tag{3.28}$$

where $\mathbb{1}$ denotes the 2×2 unit matrix and the squared physical mass of the Z boson is

$$m_Z^2 = \frac{d}{t}\left(1+\frac{1}{2}\left(h_\varphi^{(1)}+h_\varphi^{(3)}\right)\right) m'^2_Z = \frac{d}{t}\left(1+\frac{1}{2}\left(h_\varphi^{(1)}+h_\varphi^{(3)}\right)\right)\frac{g^2+g'^2}{4}v^2. \tag{3.29}$$

We remark that the simultaneous diagonalisation of the kinetic and mass terms in the neutral gauge-boson sector is completely analogous to the introduction of normal coordinates in the problem of small oscillations in mechanics; see for instance [36]. This kind of diagonalisation (3.27) has been done in [37], where the mixing term of a W_3 and a photon field is studied. A similar procedure is performed in [38] where operators up to dimension five are considered. In the approximation linear in the anomalous couplings (3.29) agrees with (4.5b) in [16] and with (4) in [17].

Similarly to the gauge bosons we now consider the terms of the Lagrangian quadratic in the Higgs field

$$\mathcal{L}_H^{(2)} = \frac{1}{2}\left(1+\frac{1}{2}\left(h_\varphi^{(1)}+h_\varphi^{(3)}\right)\right)(\partial_\mu H')(\partial^\mu H') - \frac{1}{2}m_H'^2 H'^2, \tag{3.30}$$

where $m_H'^2$ is given by (2.13). To obtain the standard form

$$\mathcal{L}_H^{(2)} = \frac{1}{2}(\partial_\mu H)(\partial^\mu H) - \frac{1}{2}m_H^2 H^2, \tag{3.31}$$

we define the physical Higgs-boson mass and physical Higgs field by a rescaling

$$m_H^2 = \frac{m_H'^2}{1+\left(h_\varphi^{(1)}+h_\varphi^{(3)}\right)/2}, \tag{3.32}$$

$$H = \sqrt{1+\left(h_\varphi^{(1)}+h_\varphi^{(3)}\right)/2}\; H'. \tag{3.33}$$

For the original Higgs-doublet field in the unitary gauge we find from (2.12) and (3.33)

$$\varphi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+\left(1+(h_\varphi^{(1)}+h_\varphi^{(3)})/2\right)^{-1/2} H(x)\end{pmatrix}. \tag{3.34}$$

For non-zero $h_\varphi^{(1)}+h_\varphi^{(3)}$ this differs from the SM result.

To analyse the phenomenology of the effective Lagrangian (2.21) we also have to express the dimension-six operators (2.15) to (2.19) in terms of the physical fields $W^\pm$, Z, A and H. In particular, we have to substitute the Higgs field according to (2.12) and (3.33). Due to (3.24), (3.25) and (3.27), the Lagrangian (2.21), and particularly the γWW, ZWW, $\gamma\gamma WW$ and $\gamma\gamma H$ vertices, depend then on the anomalous couplings in a non-linear way. We list these vertices in Sect. 6 where we treat the triple- and quartic-gauge couplings in detail.

The diagonalisation has an important consequence concerning the operators $O_{\varphi W}$ and $O_{\varphi B}$. Notice that the v^2-terms of these operators are proportional to the gauge invariant kinetic terms of the SM Lagrangian; see the first two terms of (2.3). Therefore, after the substitution of the physical fields, these operators do not give rise to anomalous three- or four-gauge-boson couplings; see Sect. 6. However, these operators contribute to the $\gamma\gamma H$ vertex.

In the next section we shall analyse the consequences of the effective Lagrangian (2.21) and of the diagonalisation (3.18) etc. for the gauge-boson–fermion couplings.

4 Gauge-boson–fermion interactions and electroweak parameters

The Lagrangian (2.21) contains the two gauge couplings g and g'. Apart from that it contains two parameters μ and λ from the Higgs potential, nine fermion masses, four parameters of the CKM matrix V, and ten anomalous couplings h_i. We can express the original parameters μ and λ in terms of m_H and v according to

$$\mu^2 = \frac{m_H'^2}{2} = \frac{1}{2}\left(1+\frac{1}{2}\left(h_\varphi^{(1)}+h_\varphi^{(3)}\right)\right)m_H^2, \tag{4.1}$$

$$\lambda = \frac{m_H'^2}{2v^2} = \frac{1}{2v^2}\left(1+\frac{1}{2}\left(h_\varphi^{(1)}+h_\varphi^{(3)}\right)\right)m_H^2. \tag{4.2}$$

We call g, g' and v the electroweak parameters. We denote the scheme that uses as input the parameters from the Lagrangian (2.21), but m_H and v instead of μ and λ, by $P_{\mathcal{L}}$; see Table 3. The quantities s'_{W}, c'_{W} and e', which are the sine and cosine of the weak mixing angle and the positron charge if we set all anomalous couplings to zero, are given in terms of the electroweak parameters in (3.3), (3.4) and (3.5), and this leads to the standard relations for the electroweak observables. However, with non-zero anomalous couplings, that is with the full Lagrangian (2.21), the relations of the three parameters g, g' and v to observables depend on the anomalous couplings.

In this section we take a look at the gauge-boson–fermion interactions and introduce two more sets of electroweak input parameters; see Table 3. In these schemes, that we call P_Z and P_W, we choose in place of g, g' and v as free parameters the fine structure constant at the

Table 3. Three parameter sets used in the analysis: $P_{\mathcal{L}}$, P_Z and P_W schemes

parameters	$P_{\mathcal{L}}$ scheme	P_Z scheme	P_W scheme
electroweak	g, g', v	$\alpha(m_Z)$, G_{F}, m_Z	$\alpha(m_Z)$, G_{F}, m_W
Higgs-boson mass	m_H	m_H	m_H
fermion masses	$m_u,\ldots, m_\tau$	$m_u,\ldots, m_\tau$	$m_u,\ldots, m_\tau$
4 CKM parameters	V	V	V
10 anomalous couplings	$h_W,\ldots, h_\varphi^{(3)}$	$h_W,\ldots, h_\varphi^{(3)}$	$h_W,\ldots, h_\varphi^{(3)}$

Z scale, $\alpha(m_Z)$, Fermi's constant G_F, and the mass of the Z or W boson, respectively. For our numerics we take

$$\begin{aligned} 1/\alpha(m_Z) &= 128.95(5), \\ G_\mathrm{F} &= 1.16639(1) \times 10^{-5}\,\mathrm{GeV}^{-2} \end{aligned} \tag{4.3}$$

from Sect. 16.3 of [41] and from [35], respectively. Moreover, from [35], we use in the P_Z scheme

$$m_Z = 91.1876(21)\,\mathrm{GeV}, \tag{4.4}$$

and in the P_W scheme

$$m_W = 80.423(39)\,\mathrm{GeV}. \tag{4.5}$$

The small errors on the quantities (4.3) to (4.5) are negligible for our purposes and will be neglected below. We use as input parameter $\alpha(m_Z)$ and not the more precisely known $\alpha(0)$, since most of the observables which we consider below refer to a high scale of at least m_Z. In the following we will denote by e the positron charge at m_Z,

$$e = \sqrt{4\pi\alpha(m_Z)}, \tag{4.6}$$

and refer to e as the physical positron charge. This is legitimate in tree-level calculations. How we include radiative corrections in our calculations will be discussed in Sect. 5 below.

We use the P_Z scheme for all LEP and SLC observables that we consider in Sect. 5. In the scheme P_Z, one can calculate the W mass m_W^SM in the SM with a certain theoretical accuracy. Using the effective Lagrangian (2.21) instead of the SM Lagrangian gives a different prediction, m_W. Indeed, as we will see in Sect. 5, two anomalous couplings have an impact on m_W in the P_Z scheme. However, for our analysis of $e^+e^- \to WW$ in Sect. 6.2 the use of the P_Z scheme with m_W depending on the anomalous couplings is very inconvenient. In [10,11] m_W is assumed to be a fixed parameter – as is legitimate and usually done in the form-factor approach – and not expanded in anomalous couplings. This is for a good reason: a change of m_W changes the kinematics of $e^+e^- \to WW$ and the reconstruction of the final state. Therefore, in Sect. 6.2 we use the P_W scheme with m_W instead of m_Z as input. In this case the Z mass is a parameter that depends on the anomalous couplings h_i.

Next we consider the fermion–gauge-boson-interaction part $\mathcal{L}_\mathrm{int}$ of the Lagrangian (2.21). Since we have not explicitly added any gauge-boson–fermion operators we get – in the original parameters – the SM expression. In terms of the fields A'_μ, Z'_μ and $W'^{\pm}_\mu$, (3.1) and (3.2), we have thus (see (22.77) and (22.123) of [20])

$$\mathcal{L}_\mathrm{int} = -e'\left(A'_\mu \mathcal{J}^\mu_\mathrm{em} + \frac{1}{s'_\mathrm{W}c'_\mathrm{W}} Z'_\mu \mathcal{J}'^\mu_\mathrm{NC} + \frac{1}{\sqrt{2}s'_\mathrm{W}}\left(W'^+_\mu \mathcal{J}^\mu_\mathrm{CC} + \mathrm{H.c.}\right)\right) \tag{4.7}$$

with the SM currents

$$\mathcal{J}^\mu_\mathrm{em} = \overline{\psi}\gamma^\mu(\mathbf{T}_3 + \mathbf{Y})\psi, \tag{4.8}$$

$$\mathcal{J}'^\mu_\mathrm{NC} = \overline{\psi}\gamma^\mu \mathbf{T}_3\psi - s'^2_\mathrm{W}\mathcal{J}^\mu_\mathrm{em}, \tag{4.9}$$

$$\mathcal{J}^\mu_\mathrm{CC} = \overline{\psi}\gamma^\mu(\mathbf{T}_1 + \mathrm{i}\mathbf{T}_2)\psi. \tag{4.10}$$

Here ψ is the spinor for all lepton and quark fields. With the mere SM Lagrangian, e' is the physical positron charge. Including the dimension-six operators we can express the interaction terms through the physical fields using (3.24) to (3.27):

$$\mathcal{L}_\mathrm{int} = -e\bigg(A_\mu \mathcal{J}^\mu_\mathrm{em} + G_\mathrm{NC} Z_\mu \mathcal{J}^\mu_\mathrm{NC} + G_\mathrm{CC}\left(W^+_\mu \mathcal{J}^\mu_\mathrm{CC} + \mathrm{H.c.}\right)\bigg), \tag{4.11}$$

where the physical positron charge (at the Z scale) is given by

$$e = \sqrt{4\pi\alpha(m_Z)} = \frac{e'}{\sqrt{d}}, \tag{4.12}$$

and the physical neutral current by

$$\mathcal{J}^\mu_\mathrm{NC} = \overline{\psi}\gamma^\mu \mathbf{T}_3\psi - s^2_\mathrm{eff}\mathcal{J}^\mu_\mathrm{em} \tag{4.13}$$

with

$$s^2_\mathrm{eff} \equiv \sin^2\theta^\mathrm{lept}_\mathrm{eff} = s'^2_\mathrm{W} + \frac{b}{d}s'_\mathrm{W}c'_\mathrm{W}. \tag{4.14}$$

The neutral- and charged-current couplings are

$$\begin{aligned} G_\mathrm{NC} &= \frac{1}{s'_\mathrm{W}c'_\mathrm{W}}\frac{d}{\sqrt{t}}, \\ G_\mathrm{CC} &= \frac{1}{\sqrt{2}s'_\mathrm{W}}\frac{\sqrt{d}}{\sqrt{1-h_{\varphi W}}}. \end{aligned} \tag{4.15}$$

The electromagnetic, the neutral- and the charged-current interactions are modified by the anomalous couplings in a universal way for fermions with the same quantum numbers. With our definition (4.14) of the effective leptonic weak mixing angle the neutral current (4.13) has the same form as in the SM, cf. (4.9). We write the neutral current as

$$\mathcal{J}^\mu_\mathrm{NC} = \sum_f \frac{1}{2}\overline{f}\left(g^f_\mathrm{V}\gamma^\mu - g^f_\mathrm{A}\gamma^\mu\gamma_5\right)f, \tag{4.16}$$

where f denotes any fermion. Then we find for the vector and axial-vector neutral-current couplings of leptons

$$g^\ell_V = 2s^2_\mathrm{eff} - \frac{1}{2}, \qquad g^\ell_A = -\frac{1}{2}, \tag{4.17}$$

with $\ell = e, \mu, \tau$. Using (4.17), we find the usual expression for s^2_eff [39]:

$$\sin^2\theta^\mathrm{lept}_\mathrm{eff} = \frac{1}{4}\left(1 - \frac{g^\ell_\mathrm{V}}{g^\ell_\mathrm{A}}\right). \tag{4.18}$$

Fermi's constant is given by two charged-current interactions in the low-energy limit where the W-boson propagator becomes point-like; see e.g. Sect. 22.3 of [20]:

$$G_\mathrm{F} = \frac{\sqrt{2}e^2}{4m_W^2}G^2_\mathrm{CC}. \tag{4.19}$$

It is related to the vacuum expectation value v of the original Higgs field φ, see (2.12), through

$$v = \left(\sqrt{2} G_{\mathrm{F}}\right)^{-1/2} \left(1 + h_\varphi^{(1)}/2\right)^{-1/2}. \tag{4.20}$$

This is obtained by inserting in (4.19) for e, G_{CC} and m_W the expressions following from (4.12), (4.15) and (3.23), respectively. For $h_\varphi^{(1)} = 0$, (4.20) becomes the tree-level SM relation between v and G_{F}. The parameter λ from the Higgs potential is therefore, cf. (4.2),

$$\lambda = \frac{G_{\mathrm{F}} m_H^2}{\sqrt{2}} \left(1 + \frac{1}{2}\left(h_\varphi^{(1)} + h_\varphi^{(3)}\right)\right)\left(1 + h_\varphi^{(1)}/2\right). \tag{4.21}$$

In the following two subsections we determine how the remaining original parameters of the Lagrangian (2.21) are related to our input parameters in the P_Z and P_W schemes. Knowing these relations one can express all constants in the Lagrangian by either of the two electroweak parameter sets plus the anomalous couplings h_i.

4.1 P_Z scheme

We now show how the original parameters in the effective Lagrangian (2.21), are expressed by the input parameters of the P_Z scheme; see Table 3. The physical Z mass m_Z and $\alpha(m_Z)$ are given in terms of the $P_{\mathcal{L}}$ parameters in (3.29) and (4.12), respectively. In the P_Z scheme the W mass m_W is a derived quantity. The relation of m_W to the $P_{\mathcal{L}}$ parameters is given in (3.23). We use (3.23), the relation $m'_W = c'_{\mathrm{W}} m'_Z$, and we express m'_Z by means of (3.29) to obtain the tree-level result for the squared W mass in the framework of the effective Lagrangian (2.21):

$$m_W^2 = \frac{t}{d} \frac{1 + h_\varphi^{(1)}/2}{\left(1 - h_{\varphi W}\right)\left(1 + \left(h_\varphi^{(1)} + h_\varphi^{(3)}\right)/2\right)} c'^2_{\mathrm{W}} m_Z^2, \tag{4.22}$$

Inserting (4.15) and (4.22) in (4.19) we obtain an equation for s'_{W}:

$$s'^2_{\mathrm{W}} = \frac{1}{2}\left\{1 - \sqrt{1 - \frac{e^2}{\sqrt{2} G_{\mathrm{F}} m_Z^2} \frac{d^2}{t} \frac{1 + \left(h_\varphi^{(1)} + h_\varphi^{(3)}\right)/2}{1 + h_\varphi^{(1)}/2}}\right\}. \tag{4.23}$$

Note that d and t contain s'_{W} and c'_{W}; see (3.13) to (3.15). Therefore (4.23) is only an implicit equation for s'_{W}, which is not easy to solve exactly. We denote the right-hand side of (4.23) for the case where all anomalous couplings are set to zero by s_0^2:

$$s_0^2 \equiv \frac{1}{2}\left(1 - \sqrt{1 - \frac{e^2}{\sqrt{2} G_{\mathrm{F}} m_Z^2}}\right), \qquad c_0^2 \equiv 1 - s_0^2. \tag{4.24}$$

Hence s_0 and c_0 are not independent parameters but combinations of input parameters in the P_Z scheme. In the SM, they are identical to the sine and cosine of the weak mixing angle. To linear order in the anomalous couplings we obtain from (4.23) in the P_Z scheme

$$s'^2_{\mathrm{W}} = s_0^2 \left(1 + c_0^2 \left(h_{\varphi W} - h_{\varphi B}\right) + \frac{4 s_0 c_0^3}{c_0^2 - s_0^2} h_{WB} + \frac{c_0^2}{2\left(c_0^2 - s_0^2\right)} h_\varphi^{(3)}\right). \tag{4.25}$$

Expanding (4.14) to first order in the couplings we find in the P_Z scheme

$$s_{\mathrm{eff}}^2 = s_0^2 \left(1 + \frac{c_0}{s_0 (c_0^2 - s_0^2)} h_{WB} + \frac{c_0^2}{2(c_0^2 - s_0^2)} h_\varphi^{(3)}\right). \tag{4.26}$$

Using (4.25) and (4.26) the quantities s'_{W}, c'_{W} and s^2_{eff} in (4.13) and (4.15) can be expressed as functions of s_0 and anomalous couplings in the linear approximation. The neutral- and charged-current couplings (4.15) read to first order in the anomalous couplings in the P_Z scheme

$$G_{\mathrm{NC}} = \frac{1}{s_0 c_0}\left(1 - \frac{1}{4} h_\varphi^{(3)}\right), \tag{4.27}$$

$$G_{\mathrm{CC}} = \frac{1}{\sqrt{2} s_0}\left(1 + \frac{s_0 c_0}{s_0^2 - c_0^2} h_{WB} + \frac{c_0^2}{4(s_0^2 - c_0^2)} h_\varphi^{(3)}\right). \tag{4.28}$$

For non-zero anomalous couplings an exact result for the W-boson mass is, in principle, obtained by inserting the solution for s'_{W} from (4.23) into (4.22). Expanding to first order in the anomalous couplings we obtain in the P_Z scheme

$$m_W = c_0 m_Z \left(1 + \frac{s_0 c_0}{s_0^2 - c_0^2} h_{WB} + \frac{c_0^2}{4\left(s_0^2 - c_0^2\right)} h_\varphi^{(3)}\right). \tag{4.29}$$

This equation is a relation at tree level. The way in which radiative corrections are taken into account in our analysis is explained at the beginning of Sect. 5. For the vacuum expectation value v we obtain to linear order in the anomalous couplings in the P_Z scheme, expanding in (4.20)

$$v = \left(\sqrt{2} G_{\mathrm{F}}\right)^{-1/2}\left(1 - h_\varphi^{(1)}/4\right). \tag{4.30}$$

4.2 P_W scheme

Similarly as in the preceding subsection we now express various quantities in the P_W scheme; see Table 3. Inserting (4.15) into (4.19) and solving for s'^2_{W} we obtain

$$s'^2_{\mathrm{W}} = \frac{e^2}{4\sqrt{2} G_{\mathrm{F}} m_W^2} \frac{d}{1 - h_{\varphi W}}. \tag{4.31}$$

Notice that in this equation d contains s'_{W} and c'_{W}. Therefore it is only an implicit equation for s'^2_{W} like (4.23). For the case where all h_i are zero the right-hand side of (4.31) is given by

$$s_1^2 \equiv \frac{e^2}{4\sqrt{2} G_{\mathrm{F}} m_W^2}, \qquad c_1^2 \equiv 1 - s_1^2. \tag{4.32}$$

Here s_1 and c_1 are combinations of input parameters of P_W. Expanding (4.31) to linear order in the anomalous couplings we obtain in the P_W scheme

$$s_{\mathrm{W}}'^2 = s_1^2 \left(1 + c_1^2 \left(h_{\varphi W} - h_{\varphi B}\right) + 2 s_1 c_1 h_{WB}\right). \qquad (4.33)$$

We expand (4.14) to first order in the h_i:

$$s_{\mathrm{eff}}^2 = s_1^2 \left(1 + \frac{c_1}{s_1} h_{WB}\right). \qquad (4.34)$$

For the neutral-current coupling (4.15) we find to first order in the anomalous couplings in P_W

$$G_{\mathrm{NC}} = \frac{1}{s_1 c_1} \left(1 + \frac{s_1}{c_1} h_{WB}\right). \qquad (4.35)$$

Here due to (4.19) and (4.32) the charged-current coupling is given exactly by

$$G_{\mathrm{CC}} = \frac{1}{\sqrt{2} s_1}, \qquad (4.36)$$

and not modified by anomalous couplings. Using the relation $m_Z' = m_W'/c_{\mathrm{W}}'$ as well as (3.23) and (3.29) we find for the squared Z mass in P_W

$$m_Z^2 = \frac{d}{t} \frac{\left(1 + \left(h_\varphi^{(1)} + h_\varphi^{(3)}\right)/2\right)\left(1 - h_{\varphi W}\right)}{1 + h_\varphi^{(1)}/2} \frac{m_W^2}{c_{\mathrm{W}}'^2}, \qquad (4.37)$$

where for s_{W}' in d and t the solution to (4.31) has to be inserted, and $c_{\mathrm{W}}' = \sqrt{1 - s_{\mathrm{W}}'^2}$. So far this is an exact expression for m_Z. To first order in the h_i the Z mass is

$$m_Z = \frac{m_W}{c_1} \left(1 + \frac{s_1}{c_1} h_{WB} + \frac{1}{4} h_\varphi^{(3)}\right). \qquad (4.38)$$

For the vacuum expectation value v to linear order in the h_i we have the same expression as in the P_Z scheme, (4.30).

5 Limits from LEP and SLC

In this section we discuss the impact of the additional operators on precision observables measured at LEP and SLC. As mentioned before we use the P_Z scheme in the entire Sect. 5. Our procedure is as follows: We calculate the tree-level prediction X_{tree} of an observable in the framework of the effective Lagrangian (2.21). Then X_{tree} can be expanded to first order in h_i

$$X_{\mathrm{tree}} = X_{\mathrm{tree}}^{\mathrm{SM}} \left(1 + \sum_i h_i \hat{X}_i\right), \qquad (5.1)$$

where $X_{\mathrm{tree}}^{\mathrm{SM}}$ is the result if we set all anomalous couplings to zero, that is the result one obtains from the tree-level calculation with the mere SM Lagrangian. At higher loop-order both X_{tree} and $X_{\mathrm{tree}}^{\mathrm{SM}}$ receive corrections. It is well known how to calculate radiative corrections in the SM; see for instance [40]. As already mentioned in the introduction radiative corrections can also be evaluated for a non-renormalisable Lagrangian like ours in (2.21) using the effective-field-theory techniques; see for instance [12]. This would result in a renormalisation of the original anomalous couplings and in the introduction of further anomalous terms of higher dimension with free coefficients. Thus, radiative corrections to our anomalous couplings should only give terms having further suppression factors α and/or (v/Λ) and will be neglected in the following. In detail, we expand the complete result X for an observable as

$$X = X^{\mathrm{SM}} \left(1 + \sum_i h_i \hat{X}_i\right) + \Delta\widetilde{X}, \qquad (5.2)$$

where X^{SM} is the complete SM result and the $\hat{X}_i$ are the *same* expressions as in (5.1). The term $\Delta\widetilde{X}$ contains then radiative corrections times and to anomalous couplings and will be neglected in the following. To get bounds on the h_i we insert the experimental values for X and use the well known higher-order results for X^{SM}. The linear parts $\hat{X}_i$ are obtained from the tree-level expansion (5.1). The experimental errors δX together with the theoretical uncertainties δX^{SM} of the SM calculation allow us then to derive bounds on the h_i. The theoretical values X^{SM} depend on the unknown Higgs mass m_H [41] and we shall discuss the bounds as functions of m_H.

As first observable we consider the leptonic mixing angle (4.14) for which we get in the P_Z scheme (4.26). There we can identify s_0 from (4.24) as the tree-level SM result

$$s_{\mathrm{eff}}^{\mathrm{SM}}\big|_{\mathrm{tree}} = s_0. \qquad (5.3)$$

According to (5.2) and (4.26) we set now

$$\begin{aligned} s_{\mathrm{eff}}^2 &= \left(s_{\mathrm{eff}}^{\mathrm{SM}}\right)^2 \left(1 + \frac{c_0}{s_0(c_0^2 - s_0^2)} h_{WB} + \frac{c_0^2}{2(c_0^2 - s_0^2)} h_\varphi^{(3)}\right) \\ &= \left(s_{\mathrm{eff}}^{\mathrm{SM}}\right)^2 \left(1 + 3.39 h_{WB} + 0.71 h_\varphi^{(3)}\right). \end{aligned} \qquad (5.4)$$

Here $s_{\mathrm{eff}}^{\mathrm{SM}}$ is the leptonic mixing angle in the SM, including radiative corrections, and the numerical values are obtained with (4.3) and (4.4).

The partial widths of the Z into a pair of fermions calculated from the Lagrangian (2.21) on tree level are

$$\begin{aligned} \Gamma_{\mathrm{f\bar{f}}}\big|_{\mathrm{tree}} &= \frac{e^2 m_Z}{48\pi} G_{\mathrm{NC}}^2 N_c^f \chi_f, \\ \chi_f &= \left(g_{\mathrm{V}}^f\right)^2 + \left(g_{\mathrm{A}}^f\right)^2, \end{aligned} \qquad (5.5)$$

where $N_c^f = 1$ for leptons and $N_c^f = 3$ for quarks. For neutrinos, charged leptons, and for up- and down-type quarks we get, respectively,

$$\chi_\nu = \frac{1}{2},$$
$$\chi_\ell = \frac{1}{2} - 2s_{\text{eff}}^2 + 4s_{\text{eff}}^4, \quad (5.6)$$
$$\chi_u = \frac{1}{2} - \frac{4}{3}s_{\text{eff}}^2 + \frac{16}{9}s_{\text{eff}}^4,$$
$$\chi_d = \frac{1}{2} - \frac{2}{3}s_{\text{eff}}^2 + \frac{4}{9}s_{\text{eff}}^4. \quad (5.7)$$

In (5.5) we have neglected all fermion masses. Setting all anomalous couplings to zero we find expressions for the tree-level partial widths in the SM as in Chapter 25 of [20]. The partial widths in (5.5) depend on the anomalous couplings through G_{NC} (4.27) and through s_{eff}^2 in χ_f. Expanding (5.5) to first order in the anomalous couplings and using our prescription (5.2), we obtain the following results for the invisible partial width, the width into one pair of charged leptons e^+e^-, $\mu^+\mu^-$ or $\tau^+\tau^-$, the hadronic and the total widths:

$$\Gamma_{\text{inv}} = \Gamma_{\text{inv}}^{\text{SM}} \left(1 - \frac{h_\varphi^{(3)}}{2}\right), \quad (5.8)$$
$$\Gamma_{\ell\ell} = \Gamma_{\ell\ell}^{\text{SM}} \left(1 + \frac{4s_0c_0(4s_0^2-1)h_{WB}}{1-6s_0^2+16s_0^4-16s_0^6} + \frac{(-1+2s_0^2+4s_0^4)\,h_\varphi^{(3)}}{2-4s_0^2(3-8s_0^2+8s_0^4)}\right), \quad (5.9)$$
$$\Gamma_{\text{had}} = \Gamma_{\text{had}}^{\text{SM}} \left(1 + \frac{4s_0c_0(44s_0^2-21)h_{WB}}{45-174s_0^2+256s_0^4-176s_0^6} + \frac{(-45+90s_0^2+4s_0^4)\,h_\varphi^{(3)}}{90-348s_0^2+512s_0^4-352s_0^6}\right), \quad (5.10)$$
$$\Gamma_Z = \Gamma_Z^{\text{SM}} \left(1 + \frac{40s_0c_0(8s_0^2-3)h_{WB}}{63-246s_0^2+400s_0^4-320s_0^6} + \frac{(-63+126s_0^2+40s_0^4)\,h_\varphi^{(3)}}{126-492s_0^2+800s_0^4-640s_0^6}\right). \quad (5.11)$$

Using (4.3), (4.4) and (4.24) we get numerically

$$\Gamma_{\text{inv}} = \Gamma_{\text{inv}}^{\text{SM}}(1 - 0.50h_\varphi^{(3)}), \quad (5.12)$$
$$\Gamma_{\ell\ell} = \Gamma_{\ell\ell}^{\text{SM}}(1 - 0.47h_{WB} - 0.60h_\varphi^{(3)}), \quad (5.13)$$
$$\Gamma_{\text{had}} = \Gamma_{\text{had}}^{\text{SM}}(1 - 1.12h_{WB} - 0.74h_\varphi^{(3)}), \quad (5.14)$$
$$\Gamma_Z = \Gamma_Z^{\text{SM}}(1 - 0.82h_{WB} - 0.67h_\varphi^{(3)}). \quad (5.15)$$

Notice that s_{eff}^2, $\Gamma_{\ell\ell}$, Γ_{had} and Γ_Z all depend on the couplings h_{WB} and $h_\varphi^{(3)}$ in a different way. In contrast, at tree level in the SM as well as with the Lagrangian (2.21) the hadronic pole cross section σ_{had}^0 as well as R_ℓ^0, R_{b}^0 and R_c^0 [41] depend only on s_{eff}^2 since they are defined in terms of ratios of the partial and total widths, such that the anomalous couplings enter only through the quantities χ_f; see (5.5) to (5.7):

$$\sigma_{\text{had}}^0 = \frac{12\pi}{m_Z^2}\frac{\Gamma_{\text{ee}}\Gamma_{\text{had}}}{\Gamma_Z^2}, \quad (5.16)$$

$$R_\ell^0 = \Gamma_{\text{had}}/\Gamma_{\ell\ell}, \qquad R_b^0 = \Gamma_{\text{b}\bar{\text{b}}}/\Gamma_{\text{had}}, \qquad R_c^0 = \Gamma_{\text{c}\bar{\text{c}}}/\Gamma_{\text{had}}. \quad (5.17)$$

Note the deviating definition of the leptonic ratio where Γ_{had} appears in the numerator. Also another group of observables, the quantities

$$\mathcal{A}_f = 2g_{\text{V}}^f g_{\text{A}}^f/\chi_f, \quad (5.18)$$

and the forward–backward asymmetries

$$A_{\text{FB}}^{0,\text{f}} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f, \quad (5.19)$$

are solely functions of s_{eff}^2:

$$\mathcal{A}_\nu = 1,$$
$$\mathcal{A}_\ell = \left(\frac{1}{2} - 2s_{\text{eff}}^2\right)/\chi_\ell, \quad (5.20)$$
$$\mathcal{A}_u = \left(\frac{1}{2} - \frac{4}{3}s_{\text{eff}}^2\right)/\chi_u,$$
$$\mathcal{A}_d = \left(\frac{1}{2} - \frac{2}{3}s_{\text{eff}}^2\right)/\chi_d. \quad (5.21)$$

We thus find that a large number of the observables listed in the summary table, Table 16.1, of [41] with the combined results from LEP1, SLC, LEP2 and W-boson measurements depend on the anomalous couplings only through s_{eff}^2, that is only through the linear combination in (5.4). These are the observables

$$\mathcal{A}_\ell(\mathcal{P}_\tau),\ \mathcal{A}_\ell(\text{SLD}),\ A_{\text{FB}}^{0,\ell},\ s_{\text{eff}}^2(\langle Q_{\text{FB}}\rangle),\ A_{\text{FB}}^{0,\text{b}},\ A_{\text{FB}}^{0,c}, \quad (5.22)$$
$$\Gamma_{\text{inv}}/\Gamma_{\ell\ell},\ R_b^0,\ R_c^0,\ \mathcal{A}_b,\ \mathcal{A}_c, \quad (5.23)$$
$$\sigma_{\text{had}}^0,\ R_\ell^0. \quad (5.24)$$

Their functional dependence on s_{eff}^2 is at tree level the same for the Lagrangian (2.21) as in the SM. Thus, neglecting again radiative corrections times and to anomalous couplings, we can use the determination of s_{eff}^2 from [41] directly for our purposes. From the six observables (5.22) the following value for s_{eff}^2 is extracted in Table 15.4 of [41]:

$$s_{\text{eff}}^2 = 0.23148 \pm 0.00017. \quad (5.25)$$

The errors of the observables (5.23) are much larger than those of the observables (5.22) and therefore do not affect this result within rounding errors, which we have checked explicitly using the tree-level expressions of the observables (5.23). Among the observables (5.22) the leptonic ones tend to give smaller values for s_{eff}^2 than the hadronic ones. This has recently been mentioned in [42]. We note that this discrepancy cannot be cured by the anomalous couplings that we consider in this paper since any choice for h_{WB} and $h_\varphi^{(3)}$ leads to one particular value of s_{eff}^2 and the observables depend on s_{eff}^2 as in the SM. For the two observables (5.24) results are given in Table 2.3 ("with lepton universality") of [41], where they are correlated with m_Z, Γ_Z and $A_{\text{FB}}^{0,\ell}$:

$$m_Z\ [\text{GeV}] = 91.1875 \pm 0.0021, \quad (5.26)$$

Table 4. Values of various observables X predicted by the SM for different Higgs masses. The dependence of their uncertainties δX on m_H is negligibly small. Taken from Figs. 15.4, 16.6 and 16.9 of [41]

m_H	120 GeV	200 GeV	500 GeV	δX
s^2_{eff}	0.23156	0.23180	0.23230	0.00030
Γ_Z [GeV]	2.4952	2.4938	2.4902	0.0026
σ^0_{had} [nb]	41.484	41.485	41.489	0.015
R^0_ℓ	20.737	20.732	20.723	0.018
m_W [GeV]	80.374	80.341	80.269	0.041
Γ_W [GeV]	2.0896	2.0880	2.0832	0.0032

$$\Gamma_Z \text{ [GeV]} = 2.4952 \pm 0.0023, \tag{5.27}$$
$$\sigma^0_{\text{had}} \text{ [nb]} = 41.540 \pm 0.037, \tag{5.28}$$
$$R^0_\ell = 20.767 \pm 0.025, \tag{5.29}$$
$$A^{0,\ell}_{\text{FB}} = 0.0171 \pm 0.0010. \tag{5.30}$$

The correlations given in the same table are, in the order m_Z, Γ_Z, σ^0_{had}, R^0_ℓ, $A^{0,\ell}_{\text{FB}}$,

$$\begin{pmatrix} 1 & -0.023 & -0.045 & 0.033 & 0.055 \\ & 1 & -0.297 & 0.004 & 0.003 \\ & & 1 & 0.183 & 0.006 \\ & & & 1 & -0.056 \\ & & & & 1 \end{pmatrix}. \tag{5.31}$$

In our scheme P_Z the Z mass is an input parameter. The forward–backward asymmetry $A^{0,\ell}_{\text{FB}}$ is already included in the result for s^2_{eff} in (5.25). We thus exclude m_Z and $A^{0,\ell}_{\text{FB}}$ from (5.26) to (5.31) by projecting the error ellipsoid onto the subspace of Γ_Z, σ^0_{had} and R^0_ℓ. Since Γ_Z depends on the couplings h_{WB} and $h^{(3)}_\varphi$ in a different way than s^2_{eff} we can in this way extract values on these two couplings from (5.25) to (5.31). The SM predictions for σ^0_{had}, R^0_ℓ and in particular for Γ_Z and s^2_{eff} depend on m_H. Their numerical values are taken from Figs. 15.4 and 16.6 of [41]. For the convenience of the reader we list these numbers in Table 4. In Table 5 we list the results for the anomalous couplings extracted from (5.25), Γ_Z, σ^0_{had} and R^0_ℓ for a Higgs mass of 120 GeV, 200 GeV and 500 GeV, respectively. The errors include the uncertainties in the SM predictions, which are mainly due to the uncertainties in $\Delta\alpha^{(5)}_{\text{had}}(m_Z^2)$, $\alpha_s(m_Z^2)$ and m_t.

We now want to include in the analysis of the anomalous couplings the data of W-mass and -width measurements. The expansion of m_W has already been given in (4.29). For the total width of the W boson we get from (4.11), (4.28) and (4.29) at tree level, neglecting fermion masses,

$$\Gamma_W|_{\text{tree}} = \frac{3e^2 m_W}{8\pi} G^2_{\text{CC}} \tag{5.32}$$
$$= \Gamma^{\text{SM}}_W\big|_{\text{tree}} \left(1 + \frac{3s_0c_0}{s_0^2 - c_0^2} h_{WB} + \frac{3c_0^2}{4\,(s_0^2 - c_0^2)} h^{(3)}_\varphi\right),$$

Table 5. Prediction of CP conserving couplings in units of 10^{-3} from the observables listed in the first row. For s^2_{eff} the result (5.25) from the observables (5.22) is used. The results are computed for a Higgs mass of 120 GeV, 200 GeV and 500 GeV, respectively. The errors δh on the couplings and the correlation between the two errors are independent of the Higgs mass within rounding errors. The correlation is −86%

	$s^2_{\text{eff}},\ \Gamma_Z,\ \sigma^0_{\text{had}},\ R^0_\ell$			
m_H	120 GeV	200 GeV	500 GeV	$\delta h \times 10^3$
$h_{WB} \times 10^3$	−0.26	−0.44	−0.68	0.81
$h^{(3)}_\varphi \times 10^3$	0.38	−0.24	−2.08	2.81

Table 6. Same as Table 5, but here m_W and Γ_W are included as observables. The correlation of the errors is −88%

	$s^2_{\text{eff}},\ \Gamma_Z,\ \sigma^0_{\text{had}},\ R^0_\ell,\ m_W,\ \Gamma_W$			
m_H	120 GeV	200 GeV	500 GeV	$\delta h \times 10^3$
$h_{WB} \times 10^3$	−0.04	−0.20	−0.43	0.79
$h^{(3)}_\varphi \times 10^3$	−1.17	−1.88	−3.81	2.39

where $\Gamma^{\text{SM}}_W\big|_{\text{tree}} = 3e^2c_0m_Z/(16\pi s_0^2)$. In the P_Z scheme the total width Γ_W depends on the same linear combination of anomalous couplings as m_W, see (4.29), and is three times more sensitive to changes of h_{WB} and $h^{(3)}_\varphi$. Now we use again our general prescription (5.2) and insert numerical values for s_0 and c_0 following from (4.3) and (4.4). We obtain then

$$m_W = m^{\text{SM}}_W(1 - 0.78h_{WB} - 0.36h^{(3)}_\varphi), \tag{5.33}$$
$$\Gamma_W = \Gamma^{\text{SM}}_W(1 - 2.35h_{WB} - 1.07h^{(3)}_\varphi). \tag{5.34}$$

We recall that in the presence of anomalous couplings all charged-current interactions are modified in a universal way. Consequently, we obtain the same relation (5.32) for all partial widths of the W boson. The branching ratios of the W boson are therefore not changed by anomalous effects, in contrast to those of the Z boson. We use the experimental values given in (16.1) and (16.2) of [41] derived from LEP, SPSC and Tevatron data

$$m_W = 80.449 \pm 0.034, \tag{5.35}$$
$$\Gamma_W = 2.136 \pm 0.069, \tag{5.36}$$

where the error correlation is −6.7%. Using the SM values for m_W and Γ_W from Fig. 16.9 of [41], which are shown in Table 4 for three different Higgs masses, and combining the bounds from m_W and Γ_W with the results from Table 5 we get the bounds on the couplings h_{WB} and $h^{(3)}_\varphi$ as listed in Table 6.

6 Three- and four-gauge-boson couplings

We now turn to the bounds on the anomalous couplings h_i from measurements of γWW and ZWW couplings at

LEP2 [41] and the prospects to measure these couplings at a future LC. The former is done in Sect. 6.1 using the scheme P_Z, the latter in Sect. 6.2 using P_W and suitably defined effective TGCs. A general parameterisation of the two triple-gauge-boson vertices by an effective Lagrangian in the ELa approach (see Sect. 1) requiring only Lorentz invariance and hermiticity consists of 14 real parameters. A common parameterisation used in the literature is the one of Hagiwara, Peccei, Zeppenfeld and Hikasa [3]:

$$\begin{aligned}\frac{\mathcal{L}_{VWW}^{\mathrm{HPZH}}}{\mathrm{i}g_{VWW}} &= g_1^V (W_{\mu\nu}^+ W^{-\mu} - W_{\mu\nu}^- W^{+\mu}) V^\nu \\ &+ \kappa_V W_\mu^+ W_\nu^- V^{\mu\nu} + \frac{\lambda_V}{m_W^2} W_{\lambda\mu}^+ W^{-\mu}{}_\nu V^{\nu\lambda} \\ &+ \mathrm{i} g_4^V W_\mu^+ W_\nu^- (\partial^\mu V^\nu + \partial^\nu V^\mu) \\ &- \mathrm{i} g_5^V \varepsilon^{\mu\nu\rho\sigma} \left(W_\mu^+ (\partial_\rho W_\nu^-) - W_\nu^- (\partial_\rho W_\mu^+) \right) V_\sigma \\ &+ \tilde{\kappa}_V W_\mu^+ W_\nu^- \tilde{V}^{\mu\nu} + \frac{\tilde{\lambda}_V}{m_W^2} W_{\lambda\mu}^+ W^{-\mu}{}_\nu \tilde{V}^{\nu\lambda}\end{aligned} \tag{6.1}$$

with $V = \gamma$ or Z. The overall constants for the photon and Z vertices are defined as follows:

$$g_{\gamma WW} = -e, \qquad g_{ZWW} = -e \cot\theta_{\mathrm{W}}, \tag{6.2}$$

where e is the positron charge. Then we have in the SM at tree level

$$g_1^V = 1, \qquad \kappa_V = 1, \tag{6.3}$$

and all other couplings equal to zero. We write $\Delta g_1^V = g_1^V - 1$ and $\Delta\kappa_V = \kappa_V - 1$ as usual. The ZWW couplings involve the mixing angle θ_{W} of the SM. In the ELa approach this θ_{W} is well defined. It is also unique at least at tree level.

Note that in the FF approach the same expression (6.1) is usually written down but allowing the coupling constants to be complex numbers. Then $\mathcal{L}_{VWW}^{\mathrm{HPZH}}$ should not be considered as an effective Lagrangian but only as a convenient shorthand description for the VWW form factors generated by using (6.1) in Feynman rules to first order. In [10, 11] the parameterisation (6.1) is used and bounds on the anomalous couplings are computed by means of optimal observables using the tree-level expressions for the differential cross section of $e^+e^- \to WW$. Given the expected accuracy at a future LC it will in general be necessary to take into account radiative corrections. How this can be done in the framework of optimal observables is explained in Sect. 3 of [10]. One can apply to the measured cross section the SM radiative corrections in the reverse to obtain a Born-level cross section. Neglecting again radiative corrections times and to anomalous couplings this Born-level cross section can be analysed using tree graphs where for the SM (6.3) is valid.

Here we want to compare the parameters h_i of our Lagrangian (2.21) – which is in the ELb approach – to the parameters in (6.1). From the outset we must make it clear that such a comparison raises problems. In the ELa approach the dimension ≤ 4 terms in the Lagrangian are exactly the SM ones. In the ELb approach investigated in the present paper on the other hand the dimension ≤ 4 terms receive anomalous contributions. The relations between the h_i and the couplings $g_1^V, \ldots, \tilde{\lambda}_V$ of (6.1) which we shall derive below are thus only valid supposing that the anomalous contributions to dimension ≤ 4 terms are negligible. For a specific process one can take into account these contributions by defining effective TGCs, as we shall do in Sect. 6.2 below for the reaction $e^+e^- \to WW$.

We now derive the relations of the parameters of (6.1) to the h_i in the approximation where terms of the Lagrangian (2.21) that are of second or higher order in h_i are neglected. The sine of the angle θ_{W} in (6.2) will be identified with s_0 in the P_Z scheme and with s_1 in the P_W scheme. The fact that we have an ambiguity here reflects again the differences of the ELa and ELb approaches.

We denote by $\mathcal{L}_{\gamma WW}$ and $\mathcal{L}_{ZWW}$ the parts of the Lagrangian (2.21) – expressed in terms of the physical fields $W_\mu^\pm$, A_μ and Z_μ – that consist of two W boson fields and one photon or Z-boson field, respectively. Without any approximation the γWW part is given by

$$\begin{aligned}\frac{\mathcal{L}_{\gamma WW}}{(-\mathrm{i}e)} &= \left(W_{\mu\nu}^+ W^{-\mu} - W_{\mu\nu}^- W^{+\mu} \right) A^\nu \\ &+ \left(1 + \frac{c'_{\mathrm{W}}}{s'_{\mathrm{W}}} \frac{h_{WB}}{(1-h_{\varphi W})} \right) W_\mu^+ W_\nu^- A^{\mu\nu} \\ &+ \frac{6\sqrt{2} G_{\mathrm{F}} s'_{\mathrm{W}}}{e\sqrt{d}} \frac{(1+h_\varphi^{(1)}/2)}{(1-h_{\varphi W})} W_{\lambda\mu}^+ W^{-\mu}{}_\nu \left(h_W A^{\nu\lambda} + h_{\tilde{W}} \tilde{A}^{\nu\lambda} \right) \\ &+ \frac{c'_{\mathrm{W}}}{s'_{\mathrm{W}}} \frac{h_{\tilde{W}B}}{(1-h_{\varphi W})} W_\mu^+ W_\nu^- \tilde{A}^{\mu\nu},\end{aligned} \tag{6.4}$$

where $\tilde{A}_{\mu\nu} = (1/2)\epsilon_{\mu\nu\rho\sigma} A^{\rho\sigma}$, and d is defined in (3.15). To obtain the term proportional to $h_{\tilde{W}}$ in (6.4) we have used the Shouten identity. Depending on whether we are in the scheme P_Z or P_W, s'_{W} is a solution to (4.23) or (4.31), respectively. The ZWW part reads

$$\begin{aligned}\frac{\mathcal{L}_{ZWW}}{(-\mathrm{i}e)} &= f_- \left(W_{\mu\nu}^+ W^{-\mu} - W_{\mu\nu}^- W^{+\mu} \right) Z^\nu \\ &+ \left(f_- - f_+ \frac{h_{WB}}{1-h_{\varphi W}} \right) W_\mu^+ W_\nu^- Z^{\mu\nu} \\ &+ \hat{f} \frac{(1+h_\varphi^{(1)}/2)}{(1-h_{\varphi W})} W_{\lambda\mu}^+ W^{-\mu}{}_\nu \left(h_W Z^{\nu\lambda} + h_{\tilde{W}} \tilde{Z}^{\nu\lambda} \right) \\ &- f_+ \frac{h_{\tilde{W}B}}{1-h_{\varphi W}} W_\mu^+ W_\nu^- \tilde{Z}^{\mu\nu},\end{aligned} \tag{6.5}$$

where $\tilde{Z}_{\mu\nu} = (1/2)\epsilon_{\mu\nu\rho\sigma} Z^{\rho\sigma}$ and

$$f_+ = \frac{1}{\sqrt{t}} \left(d + \frac{b c'_{\mathrm{W}}}{s'_{\mathrm{W}}} \right), \quad f_- = \frac{1}{\sqrt{t}} \left(\frac{d c'_{\mathrm{W}}}{s'_{\mathrm{W}}} - b \right), \tag{6.6}$$

$$\hat{f} = \frac{6\sqrt{2} G_{\mathrm{F}} s'_{\mathrm{W}}}{e\sqrt{d}} f_-. \tag{6.7}$$

Again, for the term in (6.5) proportional to $h_{\tilde{W}}$ the Shouten identity is applied. Expanding the coefficients of the operators in (6.4) and (6.5) to first order in the anomalous couplings and comparing with the Lagrangian (6.1) we find the following relations between the two sets of couplings, in the P_Z scheme:

$$\Delta g_1^\gamma = 0, \Delta\kappa_\gamma = \frac{c_0}{s_0} h_{WB}, \tag{6.8}$$

$$\Delta g_1^Z = \frac{s_0}{c_0\,(s_0^2 - c_0^2)} h_{WB} + \frac{h_\varphi^{(3)}}{4\,(s_0^2 - c_0^2)},$$
$$\Delta\kappa_Z = \frac{2 s_0 c_0}{s_0^2 - c_0^2} h_{WB} + \frac{h_\varphi^{(3)}}{4\,(s_0^2 - c_0^2)}, \tag{6.9}$$

$$\begin{aligned} \lambda_Z &= 6 s_0 c_0^2 \sqrt{2} G_\mathrm{F} m_Z^2 h_W/e, \\ \lambda_\gamma &= 6 s_0 c_0^2 \sqrt{2} G_\mathrm{F} m_Z^2 h_W/e, \end{aligned} \tag{6.10}$$

$$\tilde\kappa_Z = -\frac{s_0}{c_0} h_{\tilde W B}, \quad \tilde\kappa_\gamma = \frac{c_0}{s_0} h_{\tilde W B}, \tag{6.11}$$

$$\begin{aligned} \tilde\lambda_Z &= 6 s_0 c_0^2 \sqrt{2} G_\mathrm{F} m_Z^2 h_{\tilde W}/e, \\ \tilde\lambda_\gamma &= 6 s_0 c_0^2 \sqrt{2} G_\mathrm{F} m_Z^2 h_{\tilde W}/e, \end{aligned} \tag{6.12}$$

$$g_4^\gamma = g_4^Z = g_5^\gamma = g_5^Z = 0. \tag{6.13}$$

Equations (6.8) to (6.10) relate CP conserving couplings, whereas (6.11) and (6.12) relate CP violating ones. The couplings g_4^γ and g_4^Z are CP violating whereas g_5^γ and g_5^Z are CP conserving. From (6.8) to (6.13) we see that in our ELb framework the anomalous γWW and ZWW vertices depend only on five anomalous parameters, three of them CP conserving (h_W, h_{WB}, $h_\varphi^{(3)}$), two of them CP violating ($h_{\tilde W}$, $h_{\tilde W B}$). The 14 anomalous couplings in (6.1) thus obey 9 relations. These well known gauge relations are

$$\Delta g_1^\gamma = 0, \tag{6.14}$$

$$\Delta\kappa_Z = \Delta g_1^Z - \frac{s_0^2}{c_0^2} \Delta\kappa_\gamma, \tag{6.15}$$

$$\lambda_Z = \lambda_\gamma, \tag{6.16}$$

$$\tilde\kappa_\gamma = -\frac{c_0^2}{s_0^2} \tilde\kappa_Z, \tag{6.17}$$

$$\tilde\lambda_\gamma = \tilde\lambda_Z, \tag{6.18}$$

$$g_4^\gamma = g_4^Z = g_5^\gamma = g_5^Z = 0. \tag{6.19}$$

However, one has to keep in mind that although the number of TGCs is reduced in the ELb approach compared to the ELa approach anomalous effects can occur at other vertices or propagators; see e.g. our treatment of the reaction $e^+e^- \to WW$ in Sect. 6.2. Notice also that the gauge relations (6.14) to (6.19) do not generally hold in an $SU(2) \times U(1)$ invariant effective theory, but rather stem from the fact that we have restricted ourselves to operators of dimension ≤ 6. If one adds to the Lagrangian (2.21) suitable operators of higher dimension one can obtain a gauge invariant Lagrangian where all 14 anomalous couplings in (6.1) are independent. For this, operators up to dimension 12 are required [4], where for each additional dimension the effects are suppressed by an additional factor (v/Λ). The so-called gauge relations (6.14) to (6.19) are thus rather a low-energy approximation than a result from gauge invariance.

Using the scheme P_W, we find in the linear approximation instead of (6.8) to (6.13)

$$\Delta g_1^\gamma = 0, \quad \Delta\kappa_\gamma = \frac{c_1}{s_1} h_{WB}, \tag{6.20}$$

$$\Delta g_1^Z = 0, \quad \Delta\kappa_Z = -\frac{s_1}{c_1} h_{WB}, \tag{6.21}$$

$$\begin{aligned} \lambda_Z &= 6 s_1 \sqrt{2} G_\mathrm{F} m_W^2 h_W/e, \\ \lambda_\gamma &= 6 s_1 \sqrt{2} G_\mathrm{F} m_W^2 h_W/e, \end{aligned} \tag{6.22}$$

$$\tilde\kappa_Z = -\frac{s_1}{c_1} h_{\tilde W B}, \quad \tilde\kappa_\gamma = \frac{c_1}{s_1} h_{\tilde W B}, \tag{6.23}$$

$$\begin{aligned} \tilde\lambda_Z &= 6 s_1 \sqrt{2} G_\mathrm{F} m_W^2 h_{\tilde W}/e, \\ \tilde\lambda_\gamma &= 6 s_1 \sqrt{2} G_\mathrm{F} m_W^2 h_{\tilde W}/e, \end{aligned} \tag{6.24}$$

$$g_4^\gamma = g_4^Z = g_5^\gamma = g_5^Z = 0. \tag{6.25}$$

Notice that $h_\varphi^{(3)}$ does not enter here in P_W such that the number of couplings to describe the anomalous γWW and ZWW vertices in the P_W scheme is one less than in the P_Z scheme. We have here two CP conserving couplings (h_W, h_{WB}) and two CP violating ones ($h_{\tilde W}$, $h_{\tilde W B}$). The gauge relations (6.14) to (6.19) also hold in the scheme P_W if we substitute s_0 and c_0 by s_1 and c_1. In the P_W scheme we have a further gauge relation

$$\Delta g_1^Z = 0. \tag{6.26}$$

Thus we find in our locally $SU(2) \times U(1)$ symmetric theory that the number of independent CP conserving TGCs is three if we choose the P_Z scheme. This agrees with the results of [43]. If we choose P_W, which is actually the convenient scheme for the direct measurement of TGCs in W-boson-pair production there is one TGC less. However, the h_i also enter in fermion–boson vertices, Higgs-boson vertices and boson masses. In fact, we shall see in Sect. 6.2 that the coupling $h_\varphi^{(3)}$ affects the differential cross section of $e^+e^- \to WW$ although we use the scheme P_W.

Without approximation the $\gamma\gamma WW$ part of (2.21) is

$$\begin{aligned} \frac{\mathcal{L}_{\gamma\gamma WW}}{(-e^2)} &= \left(W_\mu^+ W^{-\mu} A_\nu A^\nu - W_\mu^+ W_\nu^- A^\mu A^\nu \right) \\ &\quad - \frac{6 s_\mathrm{W}'}{e v^2 \sqrt{d}} \frac{h_W A_{\lambda\mu} + h_{\tilde W} \tilde A_{\lambda\mu}}{(1 - h_{\varphi W})} \\ &\quad \times \Big(\left(A^\mu W_\nu^+ - A_\nu W^{+\mu} \right) W^{-\nu\lambda} + \text{H.c.} \Big). \end{aligned} \tag{6.27}$$

Using the formulae of Sect. 4 it is straightforward to calculate the linear approximation of (6.27) for the two schemes.

The terms containing two photon fields and one Higgs field in the effective Lagrangian (2.21) after diagonalisation are, without approximation,

$$\begin{aligned} & v d \sqrt{1 + \left(h_\varphi^{(1)} + h_\varphi^{(3)}\right)/2}\ \mathcal{L}_{\gamma\gamma H} \\ &= \frac{1}{2} \left(s_\mathrm{W}'^2 h_{\varphi W} + c_\mathrm{W}'^2 h_{\varphi B} - 2 c_\mathrm{W}' s_\mathrm{W}' h_{WB} \right) A_{\mu\nu} A^{\mu\nu} H \\ &\quad + \Big(s_\mathrm{W}'^2 h_{\varphi \tilde W} + c_\mathrm{W}'^2 h_{\varphi \tilde B} - c_\mathrm{W}' s_\mathrm{W}' h_{\tilde W B} \Big) \tilde A_{\mu\nu} A^{\mu\nu} H. \end{aligned} \tag{6.28}$$

Table 7. Contributions of the SM Lagrangian and of the anomalous operators to different vertices in linear order in the h_i after the simultaneous diagonalisation. Only those vertices are listed that are relevant for our observables. This does not coincide with the contributions to operators of the respective structure before the simultaneous diagonalisation; see Table 2. The coupling $h_\varphi^{(3)}$ contributes to the ZWW vertex in the scheme P_Z but not in P_W

	SM	h_W	$h_{\tilde{W}}$	$h_{\varphi W}$	$h_{\varphi \tilde{W}}$	$h_{\varphi B}$	$h_{\varphi \tilde{B}}$	h_{WB}	$h_{\tilde{W}B}$	$h_\varphi^{(1)}$	$h_\varphi^{(3)}$
γWW	√	√	√					√	√		
ZWW	√	√	√					√	√		P_Z
$\gamma\gamma WW$	√	√	√								
$\gamma\gamma H$				√	√	√	√	√	√		

In the linear approximation we simply have to drop the square root, and substitute the factor vd on the left-hand side by $(\sqrt{2}G_\mathrm{F})^{-1/2}$ and s'_W (c'_W) on the right-hand side by s_0 (c_0) in the P_Z scheme, and by s_1 (c_1) in the P_W scheme.

We summarise in Table 7 which couplings contribute to the γWW, ZWW, $\gamma\gamma WW$ and $\gamma\gamma H$ vertices if we consider only terms that are linear in the h_i.

6.1 Bounds from LEP2

For the CP conserving couplings we use the values from Table 11.7 in [41]

$$\begin{aligned} \Delta g_1^Z &= 0.051 \pm 0.032, \\ \Delta\kappa_\gamma &= -0.067 \pm 0.061, \\ \lambda_\gamma &= -0.067 \pm 0.038. \end{aligned} \tag{6.29}$$

The errors given in [41] are not symmetric. Here we make the conservative choice of taking the larger of the lower and upper errors. The correlations, in the order Δg_1^Z, $\Delta\kappa_\gamma$, λ_γ from the same reference, are

$$\begin{pmatrix} 1 & 0.23 & -0.30 \\ & 1 & -0.27 \\ & & 1 \end{pmatrix}. \tag{6.30}$$

The remaining two non-zero CP conserving couplings $\Delta\kappa_Z$ and λ_Z are not considered as independent in [41], but are assumed to be given by the gauge relations (6.15) and (6.16). From the values (6.29) and (6.30) we therefore obtain, using (6.8) to (6.10), the following values and errors for our anomalous couplings:

$$\begin{aligned} h_W &= -0.069 \pm 0.039, \\ h_{WB} &= -0.037 \pm 0.033, \\ h_\varphi^{(3)} &= -0.029 \pm 0.112, \end{aligned} \tag{6.31}$$

and the correlations, in the order h_W, h_{WB}, $h_\varphi^{(3)}$,

$$\begin{pmatrix} 1 & -0.27 & 0.36 \\ & 1 & -0.80 \\ & & 1 \end{pmatrix}. \tag{6.32}$$

We repeat that these constraints are only approximate as in our ELb framework non-SM effects do not only occur at the three-boson vertices, but also at the fermion–boson vertices and through m_W. The bounds (6.31) on the h_i are thus only valid to the approximation that these effects are negligible.[2] Moreover, in contrast to Sect. 5, no radiative corrections are included in our results here. The constraints on h_{WB} and $h_\varphi^{(3)}$ derived from TGC measurements are much weaker than the constraints from Table 6. Combining the results from Table 6 with (6.31) and (6.32) we find the values and errors as listed in Table 8. These are the final values for the CP conserving couplings that we can derive from LEP1, SLC, LEP2 and W-boson measurements. The value and error of h_W is almost independent of m_H. Electroweak data predicts a value for h_W of about -0.06. Since the errors on h_{WB} and $h_\varphi^{(3)}$ are almost uncorrelated with the error on h_W, we can consider the bounds on h_{WB} and $h_\varphi^{(3)}$ separately. Their error ellipses are shown in Fig. 1. Interestingly, a large Higgs mass is allowed by the data if h_{WB} and $h_\varphi^{(3)}$ are of order $\sim 10^{-3}$.

For the CP violating couplings we use the weighted average of the single parameter measurements given in [44,45]

$$\tilde{\lambda}_Z = 0.067 \pm 0.080, \qquad \tilde{\kappa}_Z = -0.018 \pm 0.046. \tag{6.33}$$

In these analyses the relations (6.17) and (6.18) of the CP violating photon couplings with the CP violating Z couplings are assumed to hold. Using the values (6.33) we get from (6.11) and (6.12) the results listed in Table 9. These results are independent of m_H. Since – in contrast to the CP conserving couplings – the CP violating couplings do not affect the boson–fermion couplings or the W mass these bounds are accurate in the sense that no such effects are neglected.

Bounds at 95% C.L. on anomalous TGCs have been determined by the CDF collaboration [21] and the DØ collaboration [22]. The latter, who gives the tighter

[2] In the following subsection we show that one can take into account the effects from anomalous fermion–boson couplings and anomalous boson masses by defining effective TGCs. However, to this end each physics reaction must be considered separately. Here we use the combined results from various processes and one cannot easily avoid this simplification.

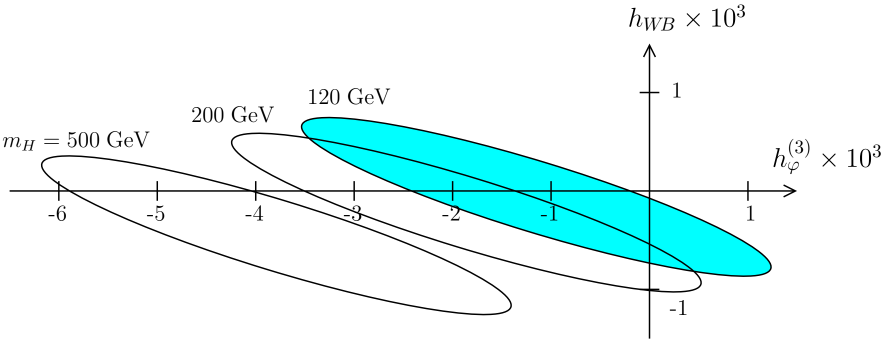

Fig. 1. Error ellipses of h_{WB} and $h_\varphi^{(3)}$ for different Higgs masses

Table 8. Final results from already existing data for CP conserving couplings in units of 10^{-3} for a Higgs mass of 120 GeV, 200 GeV and 500 GeV. The anomalous couplings are extracted from the observables listed in the first row using (5.25). The errors δh and the correlations of the errors are independent of the Higgs mass with the accuracy given here. The correlation matrix is given on the right

		s^2_{eff}, Γ_Z, σ^0_{had}, R^0_ℓ, m_W, Γ_W, TGCs						
m_H		120 GeV	200 GeV	500 GeV	$\delta h \times 10^3$			
h_W	$\times 10^3$	-62.4	-62.5	-62.8	36.3	1	-0.007	0.008
h_{WB}	$\times 10^3$	-0.06	-0.22	-0.45	0.79		1	-0.88
$h_\varphi^{(3)}$	$\times 10^3$	-1.15	-1.86	-3.79	2.39			1

Table 9. Final results from already existing data for CP violating couplings. The anomalous couplings are extracted from TGC measurements at LEP2 in various processes

	TGCs	
	h	δh
$h_{\tilde{W}}$	0.068	0.081
$h_{\tilde{W}B}$	0.033	0.084

constraints, also quotes central values and 68% C.L. limits on λ_γ and $\Delta\kappa_\gamma$. They are $\lambda_\gamma = 0.00^{+0.10}_{-0.09}$ and $\Delta\kappa_\gamma = -0.08^{+0.34}_{-0.34}$, and therefore not tighter than the constraints (6.29) from LEP2. Moreover, the values (6.29) are results where all three parameters are measured at a time. In [21] also 95% C.L. limits on two CP violating couplings are determined, viz. $-0.7 < \tilde{\lambda}_\gamma < 0.7$ and $-2.3 < \tilde{\kappa}_\gamma < 2.2$. These results can be transformed using (6.17) and (6.18) into bounds on the couplings $\tilde{\lambda}_Z$ and $\tilde{\kappa}_Z$ at 68% C.L. These resulting bounds are less stringent than the LEP2 bounds (6.33). We thus conclude that an inclusion of the bounds from [21,22] would not have a considerable effect on our calculated bounds on the h_i.

As mentioned above, see (2.23), a natural choice for the coefficients h_i in (2.22) is $h_i = \alpha_i v^2/\Lambda^2$ where Λ is the new-physics scale and the α_i are of order one. Setting $\alpha_i = 1$ and using the numbers from Tables 8 and 9 we find lower bounds Λ_i on the scale of new physics according to

$$\Lambda_i = \frac{v}{\sqrt{|h_i| + \delta h_i}}. \tag{6.34}$$

These bounds are listed in Table 10. New physics that gives rise to non-zero h_W, $h_{\tilde{W}}$ or $h_{\tilde{W}B}$ may be seen at a LC in the one-TeV-range. Those affecting $h_\varphi^{(3)}$ can lead to visible effects at a multi-TeV machine like CLIC, whereas h_{WB} will probably be out of reach in the near future. We remark that relations between the Higgs mass and the scale of new physics in an effective-Lagrangian approach have also been obtained using renormalisation group methods, see [46]. There operators of dimension six containing the Higgs and the top-quark fields are included in the effective Lagrangian, and triviality and vacuum-stability arguments are applied.

To first order in the anomalous couplings none of the observables considered so far depends on $h_{\varphi W}$, $h_{\varphi\tilde{W}}$, $h_{\varphi B}$, $h_{\varphi\tilde{B}}$ or $h_\varphi^{(1)}$. This does not change when taking into account optimal observables for $e^+e^- \to WW$ with the effective couplings; see Sect. 6.2. However, four couplings that cannot be determined with present data or in $e^+e^- \to WW$ at a future LC have an impact on the differential cross section for W-pair production at a photon collider, which we will study in a future work [34]. To be precise, one linear combination of $h_{\varphi W}$ and $h_{\varphi B}$ and one linear combination of $h_{\varphi\tilde{W}}$ and $h_{\varphi\tilde{B}}$ can be measured including data from this reaction. Then only three anomalous-coupling combinations, that is the other two linear combinations of these four couplings as well as $h_\varphi^{(1)}$,

Table 10. Lower bounds Λ_i on the new-physics scale Λ in TeV from the values of different anomalous couplings h_i obtained from the results in Tables 8 and 9 according to (6.34). The numbers are given for a Higgs mass of 120 GeV, 200 GeV and 500 GeV, respectively

m_H [GeV]	120	200	500
h_W	0.78	0.78	0.78
h_{WB}	8.4	7.7	7.0
$h_\varphi^{(3)}$	4.1	3.8	3.1
$h_{\tilde{W}}$	0.64	0.64	0.64
$h_{\tilde{W}B}$	0.72	0.72	0.72

cannot be determined. We summarise this result in Table 11 where we show which coupling combinations can be measured by means of which observables. In the right column we list all observables that we use in this work or in [34].

6.2 Effective couplings for $e^+e^- \to WW$

Here we would like to derive bounds on the anomalous couplings h_i from results obtained for the reaction $e^+e^- \to WW$ in [10,11]. There all 14 complex parameters to describe the general γWW and ZWW vertices are taken into account, see (6.1), but the fermion–boson vertices, m_Z and m_W are supposed to be as in the SM. Therefore we have to analyse carefully to which extent bounds on our anomalous couplings h_i can be obtained from [10, 11]. Consider the two cases, the ELb framework using the Lagrangian (2.21) with all anomalous couplings and the ELa framework of the Lagrangian (6.1) with only anomalous TGCs. In both cases the process $e^+e^- \to WW$ has to be calculated at tree level from three diagrams, t-channel neutrino exchange, s-channel photon and s-channel Z exchange, see Figs. 2 to 4. The various anomalous contributions in each figure are explained below. Given the projected accuracy at a future LC, it will in general be necessary to take into account radiative corrections to the process $e^+e^- \to 4$ fermions within the SM, which have been worked out in detail in the literature [47]. How these corrections can be included in an analysis with optimal observables is explained in Sect. 3 of [10]. See also the discussion after (6.3) above. In [10,11] to linear order in the anomalous TGCs the errors on their imaginary parts are not correlated with the errors on their real parts. This is because integrated observables are used and the respective anomalous amplitudes obtain different signs under the combined discrete symmetry $CP\tilde{T}$ of CP and a naïve time reversal $\tilde{T}$, that is, the simultaneous flip of all spins and momenta without interchanging initial and final state. Thus, whether or not the imaginary parts are included in the analyses of [10,11] plays no rôle when we look at the sensitivity to the real parts. For the real parts, the errors on the CP conserving couplings are not correlated with the ones on the CP violating couplings in the linear approximation, and the two groups of couplings can be considered separately [10,11]. In principle, the derivation of bounds on the h_i would require a complete calculation of the process $e^+e^- \to WW \to 4$ fermions in the framework of the Lagrangian (2.21). To first order in the couplings the errors on CP conserving and CP violating couplings are not correlated also in this case. However, in such an analysis also anomalous effects from the couplings of the Z boson to fermions, which modify the s-channel Z exchange as well as anomalous contributions to m_W (m_Z) must be taken into account if we use the scheme P_Z (P_W); see (4.29) and (4.38). Furthermore, in the scheme P_Z the anomalous couplings have an impact on the couplings of the W boson to fermions, whereas in P_W they have not due to (4.36). As mentioned in the introduction of Sect. 4, m_W is treated as a fixed parameter in [10,11]. Thus for the analysis in this section it is convenient to choose the P_W scheme. Moreover this simplifies the analysis because in P_W the neutrino-exchange amplitude contains no anomalous effects. The CP *violating* couplings appear in the reaction $e^+e^- \to WW$ only at the three-gauge-boson vertices. Thus the errors and correlations of these couplings can be obtained directly from the results in [10,11] by using (6.23) to (6.25). In contrast, in the CP *conserving* case we obtain anomalous contributions to the vertices eeZ, γWW and ZWW and to m_Z

Table 11. Anomalous couplings and observables for their measurement in the respective schemes, in which they are considered in our studies. With the ensemble of all these observables five couplings can be measured independently. In addition, of the two couplings $h_{\varphi W}$ and $h_{\varphi B}$ one linear combination can be extracted. The same is true for $h_{\varphi\tilde{W}}$ and $h_{\varphi\tilde{B}}$

P_Z scheme	
h_{WB}, $h_\varphi^{(3)}$	s^2_{eff}, Γ_Z, σ^0_{had}, R^0_ℓ, m_W, Γ_W
h_W, h_{WB}, $h_\varphi^{(3)}$	3 CP conserving TGCs
$h_{\tilde{W}}$, $h_{\tilde{W}B}$	2 CP violating TGCs
P_W scheme	
h_W, h_{WB}, $h_\varphi^{(3)}$, $h_{\tilde{W}}$, $h_{\tilde{W}B}$	effective couplings in $e^+e^- \to WW$
h_W, h_{WB}, $h_{\tilde{W}}$, $h_{\tilde{W}B}$, $(s_1^2 h_{\varphi W} + c_1^2 h_{\varphi B})$, $(s_1^2 h_{\varphi\tilde{W}} + c_1^2 h_{\varphi\tilde{B}})$	optimal observables for $\gamma\gamma \to WW$

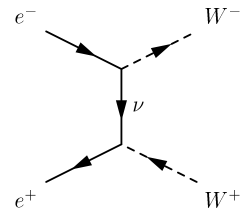

Fig. 2. Neutrino-exchange diagram

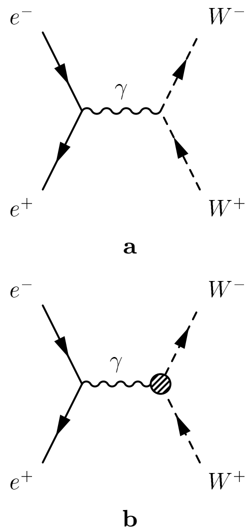

Fig. 3. Photon-exchange diagrams. SM diagram **a** and diagram with anomalous γWW couplings **b**

from the Lagrangian (2.21). Therefore in the framework of the Lagrangian (2.21), all diagrams of Figs. 2 to 4 contribute to $e^+e^- \to WW$ in zeroth or linear order in the h_i. The blobs denote anomalous couplings (without the SM contribution to the respective vertex) and the diagram (b) in Fig. 4 with the box denotes s-channel Z-boson exchange with a modified Z mass in the propagator *minus* the SM diagram, which is the diagram (a). Notice that the W-decay amplitudes remain unchanged by the h_i in the P_W scheme.

After this discussion of the calculation of the amplitude for $e^+e^- \to WW$ in our present ELb approach we compare it to the FF calculation of [10,11] which can be considered as an ELa approach if we set all imaginary parts of coupling constants there to zero. In the ELa framework of [10,11] the diagrams of Figs. 2 and 3 and only (a) and (d) of Fig. 4 occur. We will now show that the diagrams (b) and (c) of Fig. 4, that is the anomalous effects at the eeZ coupling and in m_Z, can be completely shifted to diagram (b) in Fig. 3 and diagram (d) in Fig. 4 by defining new *effective* γWW and ZWW couplings. For given values of the couplings h_i, which modify the TGCs, the fermion–boson couplings and m_Z in the ELb framework of the Lagrangian (2.21), we can compute values for these effective anomalous TGCs. Then calculating the process $e^+e^- \to WW$ in the ELa framework (6.1) of [10,11] with merely (effective) anomalous TGCs leads to the same differential cross section as calculating it with all anomalous vertices in ELb. This means the amplitudes for the process are only computed from the diagram in Fig. 2, both diagrams in Fig. 3 and diagrams (a) and (d) in Fig. 4, but with suitably defined effective γWW and ZWW couplings.

We start from the Lagrangian (2.21) and denote the parts of the amplitudes for $e^+e^- \to WW$ obtained from the tree-level diagrams for t-channel neutrino exchange, and s-channel photon and Z exchange by $\mathcal{A}_\nu$, $\mathcal{A}_\gamma$ and $\mathcal{A}_Z$, respectively. First we assume that these amplitudes are the full expressions without linearisation in the h_i. Thus these amplitudes do not correspond to the sum of the diagrams in Figs. 2 to 4, where we have assumed that all terms of second or higher order in the anomalous couplings are neglected and the diagrams with the various anomalous contributions can therefore be summed linearly. The linearisation is done in a second step below. The amplitude $\mathcal{A}_\nu$ is identical to the neutrino t-channel exchange in the SM. The amplitude $\mathcal{A}_\gamma$ is affected by the anomalous couplings only at the γWW vertex. However, we will define effective γWW couplings below because some contributions from the Z exchange will be carried over to the photon exchange. The amplitude $\mathcal{A}_Z$ is affected by anomalous couplings at the eeZ and ZWW vertices, as well as through m_Z. Now consider the currents (4.8) and (4.13) for a certain charged lepton species ℓ (in our case ℓ is the electron):

$$\mathcal{J}^\mu_{\text{em}}(\ell) = \bar{\ell}\gamma^\mu(\mathbf{T}_3 + \mathbf{Y})\ell, \tag{6.35}$$

$$\mathcal{J}^\mu_{\text{NC}}(\ell) = \bar{\ell}\gamma^\mu\mathbf{T}_3\ell - s^2_{\text{eff}}\mathcal{J}^\mu_{\text{em}}(\ell). \tag{6.36}$$

Further, we denote the vertex functions for the γWW and ZWW vertices obtained from the Lagrangian terms $\mathcal{L}_{\gamma WW}$ and $\mathcal{L}_{ZWW}$, see (6.4) and (6.5), by $\Gamma_{\gamma WW}$ and Γ_{ZWW}, respectively. They include SM as well as anoma-

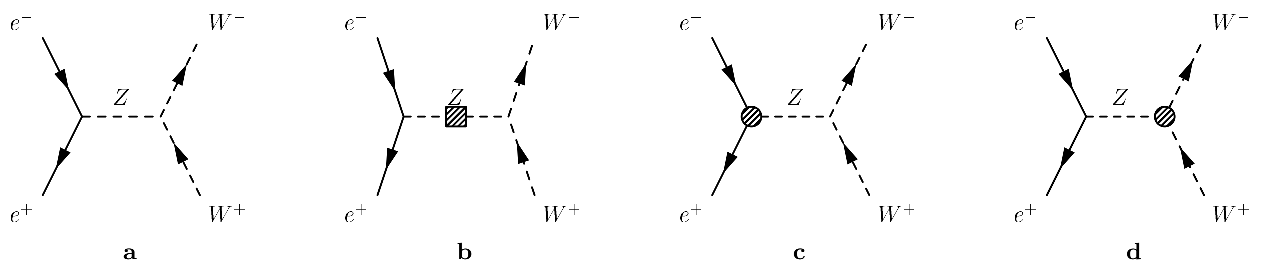

Fig. 4. Z-boson-exchange diagrams. SM diagram **a** and anomalous contributions from the modification of the Z mass **b**, from anomalous eeZ couplings **c** and anomalous ZWW couplings **d**

lous contributions, and no linear approximation in the h_i is performed yet. We have then for the sum of the amplitudes for photon and Z exchange in the P_W scheme:

$$\begin{aligned} &\mathcal{A}_\gamma + \mathcal{A}_Z \qquad (6.37)\\ &\propto \mathcal{J}^\mu_{\mathrm{em}}(\ell)\frac{1}{s}\Gamma_{\gamma WW} + G_{\mathrm{NC}}\mathcal{J}^\mu_{\mathrm{NC}}(\ell)\frac{1}{s-m_Z^2}\Gamma_{ZWW}\\ &= \mathcal{J}^\mu_{\mathrm{em}}(\ell)\frac{1}{s}\,\Gamma_{\gamma WW}|_{\mathrm{eff}}\\ &\quad + G^{\mathrm{SM}}_{\mathrm{NC}}\left(\bar{\ell}\gamma^\mu \mathbf{T}_3\ell - s_1^2\mathcal{J}^\mu_{\mathrm{em}}(\ell)\right)\frac{1}{s-\left(m_Z^{\mathrm{SM}}\right)^2}\,\Gamma_{ZWW}|_{\mathrm{eff}}\,, \end{aligned}$$

where we have defined

$$G^{\mathrm{SM}}_{\mathrm{NC}} = \frac{1}{s_1c_1}, \qquad m_Z^{\mathrm{SM}} = \frac{m_W}{c_1}, \qquad (6.38)$$

and the effective vertex functions

$$\begin{aligned} \Gamma_{\gamma WW}|_{\mathrm{eff}} &= \Gamma_{\gamma WW} \qquad (6.39)\\ &\quad + \frac{s}{s-m_Z^2}G_{\mathrm{NC}}\left(s_1^2 - s_{\mathrm{eff}}^2\right)\Gamma_{ZWW},\\ \Gamma_{ZWW}|_{\mathrm{eff}} &= \frac{G_{\mathrm{NC}}}{G^{\mathrm{SM}}_{\mathrm{NC}}}\frac{s-\left(m_Z^{\mathrm{SM}}\right)^2}{s-m_Z^2}\Gamma_{ZWW}. \qquad (6.40) \end{aligned}$$

The squared CM energy of the electron–positron system is denoted by s. From (6.37) we see that the sum of $\mathcal{A}_\gamma$ and $\mathcal{A}_Z$ can be calculated from the diagrams in Fig. 3 and diagrams (a) and (d) in Fig. 4 if we use the vertex functions $\Gamma_{\gamma WW}|_{\mathrm{eff}}$ and $\Gamma_{ZWW}|_{\mathrm{eff}}$ instead of $\Gamma_{\gamma WW}$ and Γ_{ZWW}. Expanding the coefficients of Γ_{ZWW} in (6.39) and (6.40) to linear order in the h_i we have, using (4.34),

$$\Gamma_{\gamma WW}|_{\mathrm{eff}} = \Gamma_{\gamma WW} - \frac{s}{s-m_W^2/c_1^2}h_{WB}\Gamma_{ZWW}, \quad (6.41)$$

$$\begin{aligned} &\Gamma_{ZWW}|_{\mathrm{eff}} \qquad (6.42)\\ &= \left\{1 + \frac{s_1}{c_1}\left(1+4P(s)\right)h_{WB} + P(s)h_\varphi^{(3)}\right\}\Gamma_{ZWW} \end{aligned}$$

with

$$P(s) = \frac{m_W^2/2}{c_1^2 s - m_W^2}. \qquad (6.43)$$

We can now think of $\Gamma_{\gamma WW}|_{\mathrm{eff}}$ and $\Gamma_{ZWW}|_{\mathrm{eff}}$ as vertex functions emerging from the Lagrangian terms (6.4), (6.5) and containing couplings $\Delta g_1^\gamma|_{\mathrm{eff}}$, $\Delta g_1^Z|_{\mathrm{eff}}$, etc. instead of Δg_1^γ, Δg_1^Z, etc. Taking into account the additional factor of (c_1/s_1) in the SM couplings of Γ_{ZWW} compared to the SM couplings of $\Gamma_{\gamma WW}$, see (6.1) to (6.3), we obtain to linear order in the h_i from (6.20) and (6.21)

$$\Delta g_1^\gamma|_{\mathrm{eff}} = -\frac{c_1^3}{s_1}\frac{2s}{m_W^2}P(s)h_{WB}, \qquad (6.44)$$

$$\Delta \kappa_\gamma|_{\mathrm{eff}} = -\frac{2c_1}{s_1}P(s)h_{WB}, \qquad (6.45)$$

$$\Delta g_1^Z|_{\mathrm{eff}} = \frac{s_1}{c_1}\left(1+4P(s)\right)h_{WB} + P(s)h_\varphi^{(3)}, \qquad (6.46)$$

$$\Delta \kappa_Z|_{\mathrm{eff}} = P(s)\left(\frac{4s_1}{c_1}h_{WB} + h_\varphi^{(3)}\right). \qquad (6.47)$$

With all other couplings $\lambda_\gamma|_{\mathrm{eff}}$, $\lambda_Z|_{\mathrm{eff}}$, etc. of the vertex functions $\Gamma_{\gamma WW}|_{\mathrm{eff}}$ and $\Gamma_{ZWW}|_{\mathrm{eff}}$ we drop the subscript "eff" and write λ_γ, λ_Z, etc. as usual since they are related to the h_i as before according to (6.22) to (6.25). In the high-energy limit $s \gg m_W^2$ we obtain from (6.44) to (6.47)

$$\Delta g_1^\gamma|_{\mathrm{eff}} \approx -\frac{c_1}{s_1}h_{WB}, \qquad (6.48)$$

$$\Delta \kappa_\gamma|_{\mathrm{eff}} \approx 0, \qquad (6.49)$$

$$\Delta g_1^Z|_{\mathrm{eff}} \approx \frac{s_1}{c_1}h_{WB}, \qquad (6.50)$$

$$\Delta \kappa_Z|_{\mathrm{eff}} \approx 0. \qquad (6.51)$$

The effective couplings do therefore not depend on $h_\varphi^{(3)}$ in this limit. We recall that three of the gauge relations in the P_W scheme are

$$\Delta g_1^\gamma = 0, \qquad (6.52)$$

$$\Delta g_1^Z = 0, \qquad (6.53)$$

$$\Delta \kappa_Z = \Delta g_1^Z - \frac{s_1^2}{c_1^2}\Delta\kappa_\gamma, \qquad (6.54)$$

see (6.14) and (6.15) with $s_0 \to s_1$ and $c_0 \to c_1$, and (6.26). Here, instead of these three relations we obtain two relations among the effective couplings

$$\Delta g_1^\gamma|_{\mathrm{eff}} = c_1^2\frac{s}{m_W^2}\,\Delta\kappa_\gamma|_{\mathrm{eff}}\,, \qquad (6.55)$$

$$\Delta \kappa_Z|_{\mathrm{eff}} = \Delta g_1^Z|_{\mathrm{eff}} - \frac{s_1^2}{c_1^2}\,\Delta\kappa_\gamma|_{\mathrm{eff}}\left(-2P(s)\right)^{-1}. \qquad (6.56)$$

Notice the extra factor in the brackets in (6.56) compared to the conventional relation (6.54). Instead of (6.56) one can also choose a relation, whose coefficients are energy independent:

$$\Delta \kappa_Z|_{\mathrm{eff}} = \Delta g_1^Z|_{\mathrm{eff}} - \frac{s_1^2}{c_1^2}\left(\Delta\kappa_\gamma|_{\mathrm{eff}} - \Delta g_1^\gamma|_{\mathrm{eff}}\right). \qquad (6.57)$$

However, not *both* gauge relations between the effective couplings $\Delta g_1^\gamma|_{\mathrm{eff}}$, $\Delta\kappa_\gamma|_{\mathrm{eff}}$, $\Delta g_1^Z|_{\mathrm{eff}}$ and $\Delta\kappa_Z|_{\mathrm{eff}}$ can be chosen with energy independent coefficients. This can be seen in the following way. Assume that in addition to (6.57) there is a gauge relation

$$A\,\Delta g_1^\gamma|_{\mathrm{eff}} + B\,\Delta g_1^Z|_{\mathrm{eff}} + C\,\Delta\kappa_\gamma|_{\mathrm{eff}} + D\,\Delta\kappa_Z|_{\mathrm{eff}} = 0, \qquad (6.58)$$

where A, B, C and D are constants. In the limit $s \gg m_W^2$, cf. (6.48) to (6.51), we obtain from (6.58)

$$Bs_1^2 = Ac_1^2. \qquad (6.59)$$

Now, assuming (6.58) to be independent from (6.57), we can without loss of generality set $A = 0$. Due to (6.59) we then have also $B = 0$. The relation (6.58) is then a relation solely between $\Delta\kappa_\gamma|_{\mathrm{eff}}$ and $\Delta\kappa_Z|_{\mathrm{eff}}$, which is not possible because these couplings are obviously independent, see (6.45) and (6.47). Thus no such relation (6.58) with energy independent coefficients exists. Instead at least one gauge relation, e.g. (6.55), depends on s. To summarise we obtain the following gauge relations among the effective couplings (as mentioned above for all but four couplings we drop the subscript "eff"):

$$\Delta g_1^\gamma|_{\text{eff}} = c_1^2 \frac{s}{m_W^2}\, \Delta\kappa_\gamma|_{\text{eff}}\,, \tag{6.60}$$

$$\Delta\kappa_Z|_{\text{eff}} = \Delta g_1^Z\big|_{\text{eff}} - \frac{s_1^2}{c_1^2}\, \Delta\kappa_\gamma|_{\text{eff}}\, (-2P(s))^{-1}\,, \tag{6.61}$$

$$\lambda_Z = \lambda_\gamma, \tag{6.62}$$

$$\tilde\kappa_\gamma = -\frac{c_1^2}{s_1^2}\tilde\kappa_Z, \tag{6.63}$$

$$\tilde\lambda_\gamma = \tilde\lambda_Z, \tag{6.64}$$

$$g_4^\gamma = g_4^Z \;=\; g_5^\gamma \;=\; g_5^Z \;=\; 0. \tag{6.65}$$

Instead of (6.61) one may take the relation (6.57) with energy independent coefficients.

Numerically we find from (6.22) to (6.25) that the couplings $\lambda_Z, \ldots, g_5^Z$ are expressed as linear combinations of the parameters h_i in the following way:

$$\lambda_Z = 0.980 h_W, \quad \lambda_\gamma = 0.980 h_W, \tag{6.66}$$

$$\tilde\kappa_Z = -0.544 h_{\tilde W B}, \quad \tilde\kappa_\gamma = 1.84 h_{\tilde W B}, \tag{6.67}$$

$$\tilde\lambda_Z = 0.980 h_{\tilde W}, \quad \tilde\lambda_\gamma = 0.980 h_{\tilde W}, \tag{6.68}$$

$$g_4^\gamma = g_4^Z = g_5^\gamma = g_5^Z = 0. \tag{6.69}$$

For $\sqrt{s} = 500\,\text{GeV}$ we further obtain with (6.44) to (6.47)

$$\Delta g_1^\gamma|_{\text{eff}} = -1.90 h_{WB}, \tag{6.70}$$

$$\Delta\kappa_\gamma|_{\text{eff}} = -0.064 h_{WB}, \tag{6.71}$$

$$\Delta g_1^Z\big|_{\text{eff}} = 0.582 h_{WB} + 0.017 h_\varphi^{(3)}, \tag{6.72}$$

$$\Delta\kappa_Z|_{\text{eff}} = 0.038 h_{WB} + 0.017 h_\varphi^{(3)}. \tag{6.73}$$

For $\sqrt{s} = 800\,\text{GeV}$, we have instead of (6.70) to (6.73)

$$\Delta g_1^\gamma|_{\text{eff}} = -1.86 h_{WB}, \tag{6.74}$$

$$\Delta\kappa_\gamma|_{\text{eff}} = -0.024 h_{WB}, \tag{6.75}$$

$$\Delta g_1^Z\big|_{\text{eff}} = 0.558 h_{WB} + 0.007 h_\varphi^{(3)}, \tag{6.76}$$

$$\Delta\kappa_Z|_{\text{eff}} = 0.014 h_{WB} + 0.007 h_\varphi^{(3)}. \tag{6.77}$$

In the high-energy limit $s \gg m_W^2$ we obtain from (6.48) to (6.51)

$$\Delta g_1^\gamma|_{\text{eff}} \approx -1.84 h_{WB}, \tag{6.78}$$

$$\Delta\kappa_\gamma|_{\text{eff}} \approx 0, \tag{6.79}$$

$$\Delta g_1^Z\big|_{\text{eff}} \approx 0.544 h_{WB}, \tag{6.80}$$

$$\Delta\kappa_Z|_{\text{eff}} \approx 0. \tag{6.81}$$

From the measurements of $\Delta g_1^\gamma|_{\text{eff}}$, $\Delta\kappa_\gamma|_{\text{eff}}, \ldots, g_5^Z$ in the reaction $e^+e^- \to WW$ at a future LC [10,11] we can thus get bounds on h_W, h_{WB}, $h_\varphi^{(3)}$, $h_{\tilde W}$ and $h_{\tilde W B}$ if s is not too large. In the high-energy limit $s \gg m_W^2$ the CP conserving coupling $h_\varphi^{(3)}$ cannot be measured in this way.

Table 12. Errors in units of 10^{-3} and correlations of the CP conserving couplings at CM energy $\sqrt{s} = 500\,\text{GeV}$

h	$\delta h \times 10^3$	h_W	h_{WB}	$h_\varphi^{(3)}$
h_W	0.28	1	0.09	−0.26
h_{WB}	0.32		1	−0.73
$h_\varphi^{(3)}$	36.4			1

Table 13. Same as Table 12 but for $\sqrt{s} = 800\,\text{GeV}$

h	$\delta h \times 10^3$	h_W	h_{WB}	$h_\varphi^{(3)}$
h_W	0.12	1	0.08	−0.15
h_{WB}	0.16		1	−0.79
$h_\varphi^{(3)}$	53.7			1

Table 14. Errors in units of 10^{-3} and correlations of the CP conserving couplings in the high-energy limit at CM energy $\sqrt{s} = 3\,\text{TeV}$

h	$\delta h \times 10^3$	h_W	h_{WB}
h_W	0.018	1	−0.004
h_{WB}	0.015		1

Table 15. Errors in units of 10^{-3} and correlations of the CP violating couplings at different CM energies

$\sqrt{s}$	$\delta h_{\tilde W} \times 10^3$	$\delta h_{\tilde W B} \times 10^3$	corr.
500 GeV	0.28	2.2	17%
800 GeV	0.12	1.4	9%
3 TeV	0.018	0.77	2%

6.3 Bounds from $e^+e^- \to WW$ at a linear collider

In this section we discuss the reaction $e^+e^- \to WW$, to be measured at a future linear collider, in view of its sensitivity to the anomalous couplings h_i. We assume unpolarised e^+ and e^- beams and standard expected values for the integrated luminosities [27,30] 500 fb^{-1} at $\sqrt{s} = 500\,\text{GeV}$, 1 ab^{-1} at $\sqrt{s} = 800\,\text{GeV}$ and 3 ab^{-1} at $\sqrt{s} = 3\,\text{TeV}$. We use the errors for all TGCs in the parameterisation (6.1), as given for $\sqrt{s} = 500\,\text{GeV}$ and $\sqrt{s} = 800\,\text{GeV}$ in Tables 5 and 9 of [11], respectively, and take into account their correlations (which are not listed there). We further use the corresponding results calculated for $\sqrt{s} = 3\,\text{TeV}$. From these values we can extract the errors obtainable for the h_i using (6.66) to (6.77) by conventional error propagation. We give the errors and correlations at CM energies of 500 GeV, 800 GeV and 3 TeV for the CP conserving couplings in Tables 12 to 14 and for the CP violating ones in Table 15. The errors of h_W, h_{WB}, $h_{\tilde W}$ and $h_{\tilde W B}$ at 500 GeV are considerably smaller than the one on $h_\varphi^{(3)}$. Notice that $h_\varphi^{(3)}$ becomes unmeasurable in the high-energy limit; see (6.78) to (6.81). At $\sqrt{s} = 3\,\text{TeV}$ we thus obtain no bound on $h_\varphi^{(3)}$. For all other measurable couplings the errors become much smaller with rising energy. Notice that the error correlations decrease with rising energy and the four measurable couplings are almost uncorrelated at $\sqrt{s} = 3\,\text{TeV}$.

7 Conclusions

We have analysed the phenomenology of the gauge-boson sector of an electroweak locally $SU(2) \times U(1)$ invariant effective Lagrangian. In addition to the SM Lagrangian we took into account anomalous coupling terms from the ten operators of dimension six built either only from the SM gauge fields or from the SM gauge fields combined with the SM-Higgs-doublet field. We found that after SSB some anomalous terms contribute to the diagonal and off-diagonal kinetic terms of the neutral gauge bosons, to the mass terms of the W and the Z bosons, and to the kinetic term of the Higgs boson. This made necessary to first identify the physical neutral gauge-boson fields as linear combinations of the fields that originally occur in the Lagrangian, and to renormalise the Higgs-boson field and the charged gauge-boson fields. In this way, in addition to the gauge-boson self-interactions, also the neutral- and charged-current interactions were modified. A careful discussion of electroweak parameterisation schemes was given; see Table 3. We have studied the impact of anomalous couplings onto LEP and SLC observables. For a large class of observables the anomalous effects only show up through a modified effective leptonic weak mixing angle; see Sect. 5. The functional dependence of these observables on the effective mixing angle is the same as in the SM. Thus the discrepancy between the predictions for this angle from hadronic and leptonic observables cannot be obtained by non-zero anomalous couplings from our boson operators. The observables Γ_Z, m_W and Γ_W, depend on the anomalous couplings in a different way and therefore lead to further constraints. From all these observables we obtain bounds of order 10^{-3} for the dimensionless couplings h_{WB} and $h_\varphi^{(3)}$. These bounds depend on m_H.

Turning then to the TGCs we found that in addition to the two couplings h_{WB} and $h_\varphi^{(3)}$ one more CP conserving coupling, h_W, and the two CP violating couplings $h_{\tilde{W}}$ and $h_{\tilde{W}B}$ modify the γWW and ZWW vertices in the scheme P_Z. In the scheme P_W the triple-gauge-boson vertices are parameterised by one coupling less than in P_Z; see Tables 3 and 7. In other words there is an additional gauge relation in the scheme P_W. However, both with P_Z and with P_W some CP conserving couplings also change the boson–fermion interactions. For the specific reaction $e^+e^- \to WW$ and using P_W we have defined effective TGCs such that all anomalous effects are absorbed into the effective three-gauge-boson vertices $\Gamma_{\gamma WW}|_{\text{eff}}$ and $\Gamma_{ZWW}|_{\text{eff}}$. The anomalous gauge-boson–fermion interactions are thus fully taken into account here (in the approximation linear in the h_i) though in the explicit calculation of the differential cross section everything apart from the TGCs is assumed to be SM like. With the effective couplings one more parameter re-enters the differential cross section in the scheme P_W. The gauge relations between the effective couplings are different from those between standard TGCs. At least one gauge relation contains the squared CM energy s of the electron–positron system.

For the bounds derived from LEP2 data that includes various processes and not only W-boson-pair production we have used P_Z and only considered the conventional TGCs. This gives exact results for the CP violating couplings, but only approximate results for the CP conserving ones, since we have neglected the modified W mass and boson–fermion interactions there. For the couplings h_{WB} and $h_\varphi^{(3)}$ the direct LEP2 measurements do not give tighter bounds than the other LEP and SLC observables. However, we obtain in addition bounds on h_W, $h_{\tilde{W}}$ and $h_{\tilde{W}B}$ of order 0.1.

Our summary of the presently available information on the anomalous couplings h_i is presented in Tables 8 and 9 and in Fig. 1. We find that the data is consistent with a light Higgs boson, $m_H = 120\,\text{GeV}$ and practically vanishing anomalous couplings. But also a heavy Higgs boson, $m_H \approx 500\,\text{GeV}$, is in accordance with the present data if only small anomalous couplings h_{WB} and $h_\varphi^{(3)}$ of order 10^{-3} are introduced in the gauge-boson sector; see Fig. 1. Moreover the data prefer a value for h_W of -0.06 over $h_W = 0$ at the 2σ level; see Table 8. This may change if radiative corrections are included in the relevant LEP2 analyses of TGCs.

We have investigated in detail the effects of our effective Lagrangian on the reaction $e^+e^- \to WW$ at a future LC. To this end we have used the results obtained for solely TGCs in the most general parameterisation for unpolarised beams and longitudinal polarisation [10] as well as for transverse polarisation [11]. These analyses have been done with optimal observables and the derived constraints on the h_i therefore give the optimal bounds that one can obtain in this reaction from the normalised event distribution. Here we have used the scheme P_W and our technique with the effective vertices $\Gamma_{\gamma WW}|_{\text{eff}}$ and $\Gamma_{ZWW}|_{\text{eff}}$. For most couplings the bounds obtainable with standard expected integrated luminosities are δh_i around a few 10^{-4} to 10^{-3} at a CM energy $\sqrt{s} = 500\,\text{GeV}$ and are greatly improved with rising energy. Only one coupling, $h_\varphi^{(3)}$, is not measurable in the high-energy limit.

Now we compare our results to the ones of [18,19]. The authors of [18] have calculated at tree and one-loop level the $\gamma\gamma$-, γZ-, ZZ- and WW-two-point functions as well as the vector-boson–fermion vertex functions in an effective Lagrangian approach with two additional operators of dimension six. Thus they are more general in considering also loop effects but in the present work we are more general in including more operators.

In the extensive work [19] a gauge invariant effective Lagrangian with dimension-six operators is considered. There only C and P conserving operators are included. The total set of operators that can be constructed using the gauge fields and the Higgs field of the SM is reduced by discarding terms which are only total derivatives. However, in contrast to [16] and to our analysis here, the equations of motion are not applied in the reduction of the number of operators since the authors of [19] considered tree-level and one-loop effects. Compared to this work we have studied here the tree-level effects of C, P and CP conserving and violating operators. We have also shown the advantages and disadvantages of the two parameterisation schemes, P_Z and P_W, for the study of TGCs. Finally we have defined effective TGCs in the scheme P_W which allow a direct comparison of the ELb, ELa and FF approaches for $e^+e^- \to WW$.

An extensive study [23] has discussed the measurement of the γWW and ZWW couplings at the LHC. Using events with a $W^\pm Z$ ($W^\pm\gamma$) pair in the final state one is sensitive only to the ZWW (γWW) couplings in Drell–Yan-type production, and therefore the two groups of couplings can be measured separately. Since our results are in the ELb framework they cannot be directly compared to those of [23] where merely anomalous TGCs are assumed. In [23] bounds on three TGCs from events with a $W^\pm Z$ *or* a $W^\pm\gamma$ pair are also computed in a framework with a gauge invariant effective Lagrangian. However there, too, effects from other vertices or propagators are not considered and therefore also these results cannot be directly compared with ours. For all these reasons we conclude that a concise comparison of the sensitivity at the LHC with the bounds from a future LC calculated in this paper requires a full calculation of the processes there, which is beyond the scope of the present work. We should also note that the TGCs studied for the LHC in [23] the ZWW and γWW vertices are studied for one W far off-shell, the other W and the Z and γ on-shell. In our LC study the two Ws are on-shell, the Z and γ far off-shell. We see that there is nice complementarity of the LHC and LC possibilities.

Coming back to the results of our present paper we note that the Giga-Z mode at TESLA, see Sect. 5.1.4 of [28], will be particularly interesting to accurately measure h_{WB} and $h_\varphi^{(3)}$. A measurement at the Z pole with an event rate that is about 100 times that of LEP1, should in essence reduce the errors δh given in Table 5 by a factor 10. Thus h_{WB} and $h_\varphi^{(3)}$ can then be measured with an accuracy of some 10^{-4}. However, systematical errors can become more important there [48].

A very interesting opportunity for the exploration of the electroweak gauge-boson sector is the measurement of the differential cross section of $\gamma\gamma \to WW$ at a photon collider, which we shall explore in a future work [34]. Here two new coupling combinations can be determined that cannot be measured with the other options that we have considered.

We have seen that experiments performed in the past as well as the Giga-Z, the e^+e^- and the $\gamma\gamma$ options at a future LC all provide and will provide useful and complementary information on the gauge-boson sector. At present a non-zero value is preferred for h_W at the 2σ level, while small h_{WB} and $h_\varphi^{(3)}$ can make a heavy standard model Higgs boson with $m_H \approx 500\,\mathrm{GeV}$ compatible with the data. The bounds on the CP conserving anomalous couplings depend on the mass of the Higgs boson. Until the Higgs boson is found the bounds on these couplings can therefore only be given as a function of m_H. If a Higgs boson is discovered at the LHC the constraints on the CP conserving couplings from LEP and SLC observables can be precisely stated. The present bounds on the CP violating couplings are rather loose. In the future, with data from all three mentioned linear collider modes seven out of ten anomalous coupling combinations can be measured. Our study in this paper and the one to follow on the reaction $\gamma\gamma \to WW$ should make it clear that exploring the electroweak gauge structure needs a comprehensive study at a future linear collider where all running modes are needed and will reveal interesting complementary aspects.

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